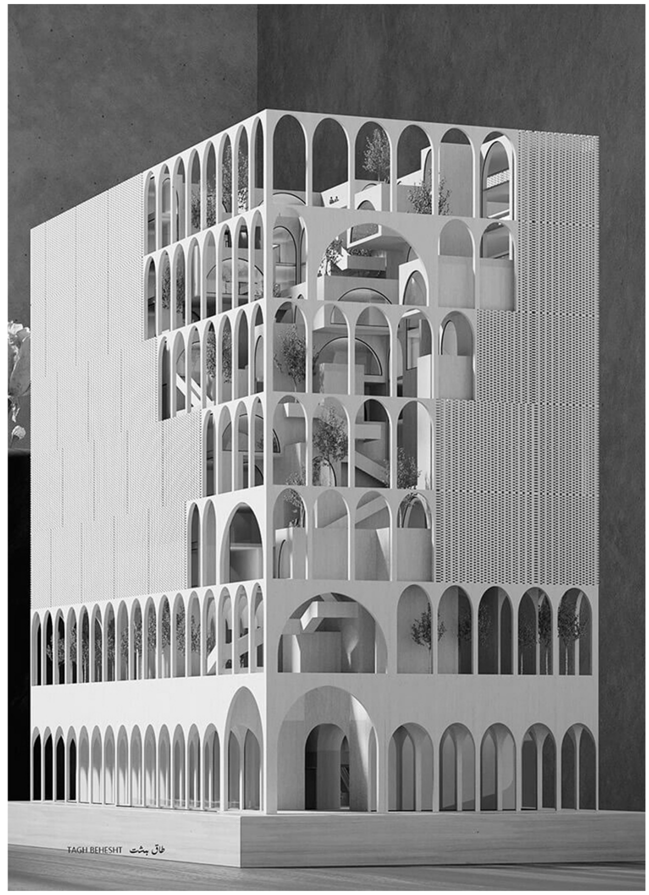
Architecture 504
Masonry Structures

Combined Axial and Flexure Load

Part 2

- Concentric axial
- Interaction
- Bearing walls



Tagh Behesht
by Rvad Studio

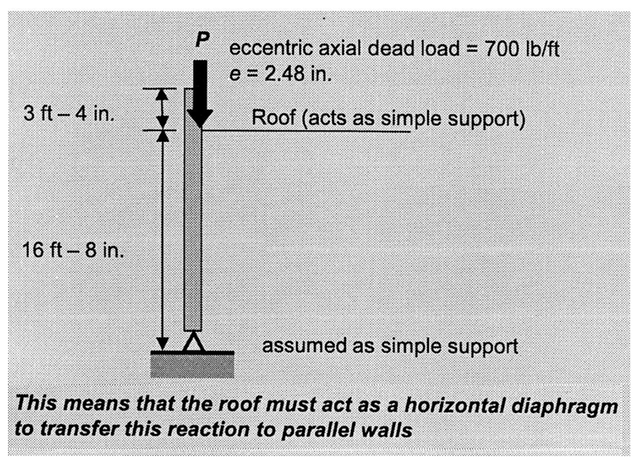
Tagh Behesht by Rvad Studio

The project's primary design methodology began with an investigation of architectural history of bazaars in Iran and the city of Mash-had. Since time immemorial, the unbreakable bond between the city bazaars and the foundations of the economy has led to bazaars taking on an important and consistent role in people's daily lives.

Tagh Behesht by Rvad Studio

Combined Bending and Axial Load example

Given:
- exterior wall w/ parapet
- 30 psf wind
- eccentric roof, e=2.48”
- wall DL = 44 psf
- Gr. 60 steel
- assume grout 48” o.c. (6 cells)
- 8” CMU w/ type S masonry cement
- load combination: 0.9D + 1.0W



Required:
- Find the required steel reinforcement

Procedure:
1. Calculate the initial (without magnifier) Mu and Pu
2. Calculate the moment magnifier, ψ
3. Determine the revised Mu and Pu
4. Determine combined force mode – tension or compression controlled
5. Find required steel, $A_{s,reqd}$

Combined Bending and Axial Load example

Procedure:

1. Calculate the initial (without magnifier) Mu and Pu
2. Calculate the moment magnifier, ψ
3. Determine the revised Mu and Pu
4. Determine combined force mode – tension or compression controlled
5. Find required steel, $A_{s,reqd}$

Calculate

$$a = d - \sqrt{d^2 - \frac{2[P_u(d - t_{sp}/2) + M_u]}{\phi(0.8 f'_m b)}} \qquad c = \frac{a}{0.8}$$

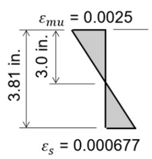
Is $c \geq c_{bal}$?

For CMU, Grade 60 steel

$c_{bal} = 0.547d$

$$c_{bal} = \frac{\varepsilon_{mu}}{\varepsilon_{mu} + \varepsilon_y} d$$

YES



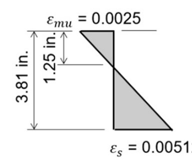
$$A_{s,reqd} = \frac{0.8 f'_m b a - P_u/\phi}{\varepsilon_{mu} E_s \left(\frac{d - c}{c}\right)}$$

Compression controls

NO

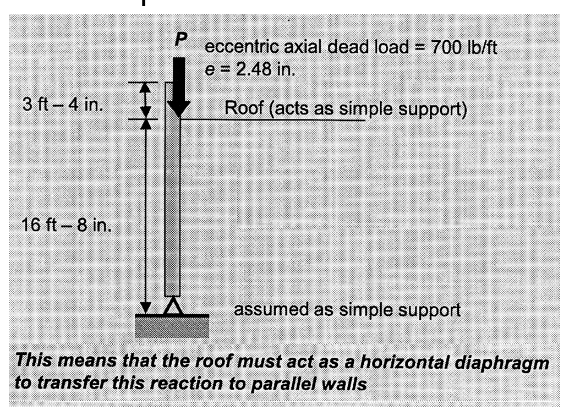
$$A_{s,reqd} = \frac{0.8 f'_m b a - P_u/\phi}{f_y}$$

Tension controls



Combined Bending and Axial Load example

1. Calculate the initial (without magnifier) Mu and Pu



Calculate original moment, Mu,o (without ψ)

The maximum moment will occur approximately at midheight of the wall, and can be determined as:

$$M_{u,o} = \frac{w_u h^2}{8} + \frac{P_{uf} e_u}{2} - \frac{1}{2}\frac{w_u h^2_{parapet}}{2}$$

$$= \left[\frac{(30 \text{ psf})(16.67 \text{ ft})^2}{8} + \frac{0.9(700 \text{ lb/ft})(2.48 \text{in.}/12 \text{ ft})}{2} - \frac{1}{2}\frac{(30 \text{ psf})(3.33 \text{ ft})^2}{2}\right]\frac{12 \text{ in.}}{\text{ft}}$$

$$= 13{,}100 \text{ lb} - \text{in./ft}$$

Calculate Pu

The axial load at midheight is:

$$P_u = 0.9D = 0.9\left(700 \text{ lb/ft} + 44 \text{ psf}\left(3.33 \text{ ft} + 16.67 \text{ ft}/2\right)\right) = 1{,}090 \text{ lb/ft}$$

Combined Bending and Axial Load example

2. Find the moment magnification factor, ψ

Calculate Mcr and compare with Mu (estimate ψ = 1.1)

Mu = 1.1(13100) = 14410 in-lb/ft
Mcr = 8260 in-lb/ft
therefore, section is cracked

9.3.5.4.3 The strength level moment, M_u, shall be determined either by a second-order analysis, or by a first-order analysis and Equations 9-27 through 9-29.

$$M_u = \psi M_{u,0} \qquad \text{(Equation 9-27)}$$

Where $M_{u,0}$ is the strength level moment from first-order analysis.

$$\psi = \frac{1}{1 - \frac{P_u}{P_e}} \qquad \text{(Equation 9-28)}$$

Where:

$$P_e = \frac{\pi^2 E_m I_{eff}}{h^2} \qquad \text{(Equation 9-29)}$$

For $M_u < M_{cr}$, I_{eff} shall be taken as $0.75 I_n$. For $M_u \geq M_{cr}$, I_{eff} shall be taken as I_{cr}. P_u/P_e cannot exceed 1.0.

Cracking Moment:

Type S masonry cement, tension normal to the bed joint, TMS 402 Table 9.1.9.2

Ungrouted: $f_r = 51$ psi   Grouted: $f_r = 153$ psi

Modulus of rupture (linear interpolation):

$$\left(\frac{5 \text{ ungrouted cells}}{6 \text{ cells}}\right) 51 \text{ psi} + \left(\frac{1 \text{ grouted cell}}{6 \text{ cells}}\right) 153 \text{ psi} = 68 \text{ psi}$$

Cracking Moment: $$M_{cr} = \left(\frac{P_u}{A_n} + f_r\right) S_n = \left(\frac{1{,}090 \text{ lb/ft}}{40.7 \text{ in.}^2\text{/ft}} + 68 \text{ psi}\right) 87.1 \text{ in.}^3\text{/ft} = 8{,}260 \text{ lb-in./ft}$$

Combined Bending and Axial Load example

2. Find the moment magnification factor, ψ

Calculate Icr = Ieff

$$I_{cr} = n\left(A_s + \frac{P_u}{f_y}\frac{t_{sp}}{2d}\right)(d-c)^2 + \frac{bc^3}{3} \qquad \text{(Equation 9-30)}$$

$$c = \frac{A_s f_y + P_u}{0.64 f'_m b} \qquad \text{(Equation 9-31)}$$

Cracked Moment of Inertia:

Modular ratio: $$n = \frac{E_s}{E_m} = \frac{29{,}000{,}000 \text{ psi}}{900(2{,}000 \text{ psi})} = \frac{29{,}000{,}000 \text{ psi}}{1{,}800{,}000 \text{ psi}} = 16.11$$

Depth to neutral axis: $$c = \frac{A_s f_y + P_u}{0.64 f'_m b} = \frac{0.05 \text{ in.}^2\text{/ft}(60{,}000 \text{ psi}) + 1{,}090 \text{ lb/ft}}{0.64(2{,}000 \text{ psi})(12 \text{ in./ft})} = 0.266 \text{ in.}$$

Cracked moment of inertia: $$I_{cr} = n\left(A_s + \frac{P_u}{f_y}\frac{t_{sp}}{2d}\right)(d-c)^2 + \frac{bc^3}{3}$$

$$= 16.11\left(0.05 \text{ in.}^2\text{/ft} + \frac{1{,}090 \text{ lb/ft}}{60{,}000 \text{ psi}}(1)\right)(3.81 \text{ in.} - 0.266 \text{ in.})^2 + \frac{(12 \text{ in./ft})(0.266 \text{ in.})^3}{3} = 13.9 \text{ in.}^4\text{/ft}$$

Combined Bending and Axial Load example

2. Calculate the moment magnification factor, ψ

$$\psi = \frac{1}{1 - \frac{P_u}{P_e}} \quad \text{(Equation 9-28)}$$

Where:

$$P_e = \frac{\pi^2 E_m I_{eff}}{h^2} \quad \text{(Equation 9-29)}$$

Calculate Pe

Buckling load: $P_e = \frac{\pi^2 E_m I_{eff}}{h^2} = \frac{\pi^2 (1{,}800{,}000 \text{ psi})(13.9 \text{ in.}^4/\text{ft})}{(200 \text{ in.})^2} = 6{,}170 \text{ lb/ft}$

Combined Bending and Axial Load example

2. Calculate the moment magnification factor, ψ

$$\psi = \frac{1}{1 - \frac{P_u}{P_e}} \quad \text{(Equation 9-28)}$$

Where:

$$P_e = \frac{\pi^2 E_m I_{eff}}{h^2} \quad \text{(Equation 9-29)}$$

Calculate ψ

Moment magnifier: $\psi = \frac{1}{1 - \frac{P_u}{P_e}} = \frac{1}{1 - \frac{1{,}090 \text{ lb/ft}}{6{,}170 \text{ lb/ft}}} = 1.214$

3. Determine the revised Mu and Pu

Mu = ψ Mu,0 = 1.214(13100) = 15903 in-lb/ft

Pu = 0.9 DL = 0.9 (floor + parapet + wall/2) = 1090 lb/ft

The axial load at midheight is:

$$P_u = 0.9D = 0.9\,(700 \text{ lb/ft} + 44 \text{ psf}\,(3.33 \text{ ft} + 16.67 \text{ ft}/2)) = 1{,}090 \text{ lb/ft}$$

Combined Bending and Axial Load example

4. Determine combined force mode: tension or compression controlled?

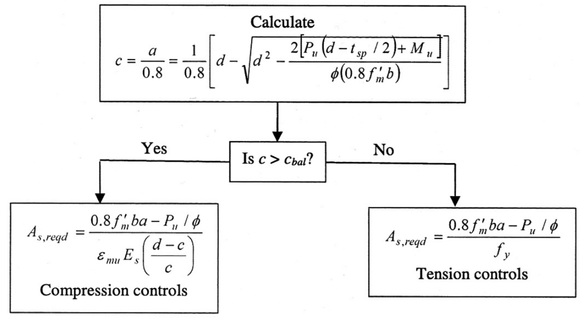


Figure 12.4-4 Flow Chart for Strength Design of Flexure and Axially Loaded Members

Determine c:

a = 0.2497 in.

c = a/0.8 = 0.312 in.

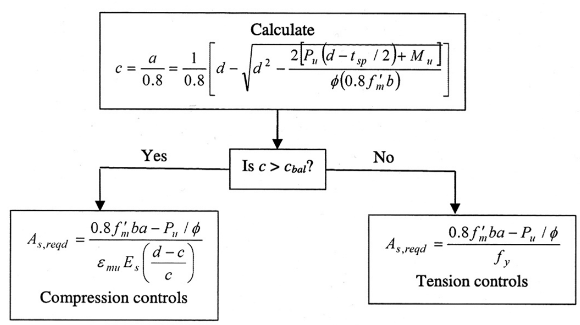
For centered reinforcement, which is often the case in walls, the axial force does not affect the value of c since $d = t_{sp}/2$ and thus $d\text{-}t_{sp}/2 = 0$.

$$c=\frac{1}{0.8}\left[d-\sqrt{d^2-\frac{2\left[P_u\left(d-t_{sp}/2\right)+M_u\right]}{\phi\left(0.8f'_m b\right)}}\right]$$

$$=\frac{1}{0.8}\left[3.81\,\text{in.}-\sqrt{(3.81\,\text{in.})^2-\frac{2\left[15903\,\text{lb-in./ft}\right]}{0.9(0.8)(2{,}000\,\text{psi})(12\,\text{in./ft})}}\right]=0.312\,\text{in.}$$

Combined Bending and Axial Load example

4. Determine combined force mode: tension or compression controlled?



Determine c balanced

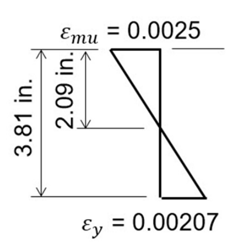
Figure 12.4-4 Flow Chart for Strength Design of Flexure and Axially Loaded Members

c_{bal} = d 0.547

= (3.81) 0.547 = 2.084 in.

Balance Point

Location of neutral axis: $c=d\left(\frac{\varepsilon_{mu}}{\varepsilon_{mu}+\varepsilon_y}\right)=3.81\,\text{in.}\left(\frac{0.0025}{0.0025+0.00207}\right)=2.08\,\text{in.}$



Strain

c = 0.312 in. < c_{bal} = 2.084 in.

c < c_{bal} therefore, tension controls

Combined Bending and Axial Load example

10. Determine $A_{s,reqd}$

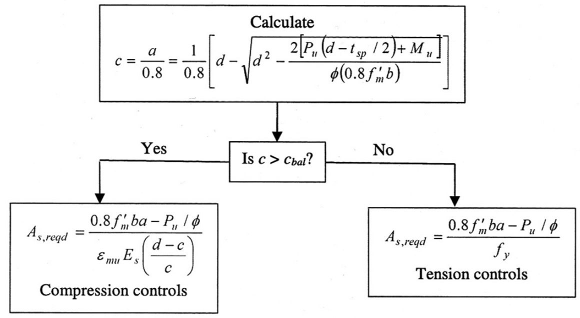


Figure 12.4-4 Flow Chart for Strength Design of Flexure and Axially Loaded Members

$$A_{s,reqd} = \frac{0.8 f'_m ba - P_u/\phi}{f_y}$$

$$= \frac{0.8(2{,}000 \text{ psi})(12 \text{ in./ft})(0.249 \text{in.}) - (1{,}090 \text{ lb/ft})/0.9}{60{,}000 \text{ psi}} = 0.0597 \text{ in.}^2/\text{ft}$$

Combined Bending and Axial Load example

for A_{sreqd} = 0.597 in^2/ft

use #4 @ 40 in. o.c.

from TEK 14-01B

An = 42.8 in^2/ft

Sn = 88.3 in^3/ft

check Mcr

check capacity

check ρ max

Spacing (inches)	Steel Area in.^{2}/ft			
	#3	#4	#5	#6
8	0.16	0.30	0.46	0.66
16	0.082	0.15	0.23	0.33
24	0.055	0.10	0.16	0.22
32	0.041	0.075	0.12	0.16
40	0.033	0.060	0.093	0.13
48	0.028	0.050	0.078	0.11
56	0.024	0.043	0.066	0.094
64	0.021	0.038	0.058	0.082
72	0.018	0.033	0.052	0.073
80	0.016	0.030	0.046	0.066
88	0.015	0.027	0.042	0.060
96	0.014	0.025	0.039	0.055
104	0.013	0.023	0.036	0.051
112	0.012	0.021	0.033	0.047
120	0.011	0.020	0.031	0.044

Combined Bending and Axial Load example

for A_s = 0.6 in^2/ft

use #4 @ 40 in. o.c.

from TEK 14-01B

An = 42.8 in^2/ft

Sn = 88.3 in^3/ft

$$a = \frac{A_s f_y + P_u/\phi}{0.80 f'_m b} = \frac{(0.06\text{ in.}^2/\text{ft})(60{,}000\text{ psi}) + (1{,}090\text{ lb/ft})/0.9}{0.80(2{,}000\text{ psi})(12\text{ in./ft})} = 0.250\text{ in.}$$

$$M_n = (P_u/\phi + A_s f_y)\left(d - \frac{a}{2}\right)$$

$$= ((1{,}090\text{ lb/ft})/0.9 + (0.06\text{ in.}^2/\text{ft})(60{,}000\text{ psi}))\left(3.81\text{ in.} - \frac{0.250\text{ in.}}{2}\right) = 17{,}729\text{ lb - in./ft}$$

$$\phi M_n = 0.9(15{,}600\text{ lb - in./ft}) = 15{,}956\text{ lb - in./ft}$$

φMn = 15,956 in-lb/ft > 15,903 in-lb/ft = Mu ok

check capacity

check ρ max

(TMS 402 9.3.3.2)

$$\rho_{max} = \frac{0.64 f'_m\left(\frac{\varepsilon_{mu}}{\varepsilon_{mu} + 1.5\varepsilon_y}\right) - \frac{P}{bd}}{f_y}$$

$$= \frac{0.64(2{,}000\text{ psi})\left(\frac{0.0025}{0.0025 + 1.5(0.00207)}\right) - \frac{700\text{ lb/ft}}{(12\text{ in./ft})(3.81\text{ in.})}}{60{,}000\text{ psi}} = 0.00926$$

The actual reinforcement ratio is:

$$\rho = \frac{A_s}{bd} = \frac{0.06\text{ in.}^2/\text{ft}}{(12\text{ in./ft})(3.81\text{ in.})} = 0.00131 \le 0.00926 \quad \text{OK}$$

Combined Bending and Axial Load

CMU Pilaster Design

h = 24 ft

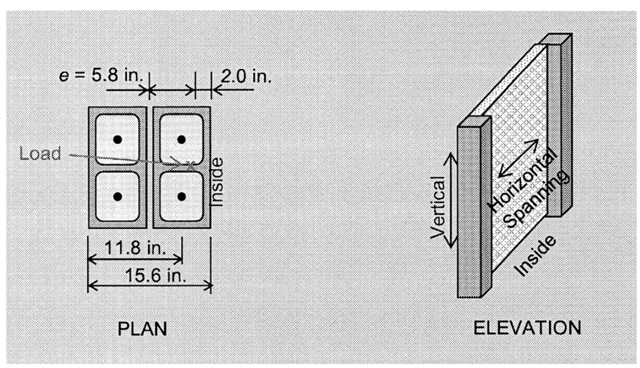
o.c. spacing = 16 ft

f'm = 2000 psi

Gr 60 reinforcement

reinforcement not laterally tied

eccentricity = 5.8 in.



D = 9.6 k S = 9.6 k W = 8.1 k (uplift) W = 26 psf (lateral)

<u>Loads:</u>

Check load combination $0.9D + 1.0W$ as it usually controls.

Pilaster weight (fully grouted): 75 psf(1.33 ft)(2 wythes) = 200 lb/ft

Out-of-plane wind load on pilaster: (26 psf)(16 ft) = 416 lb/ft

Axial load at top of pilaster, P_{uf}: 0.9(9,600 lb) – 1.0(8,100 lb) = 540 lb

Axial load at midheight: $P_u = P_{uf} + P_{u,pilaster}$ = 540 lb + 0.9 (200 lb/ft)(12 ft) = 2,700 lb

The maximum moment will occur approximately at the midheight of the pilaster.

Combined Bending and Axial Load

CMU Pilaster Design

h = 24 ft

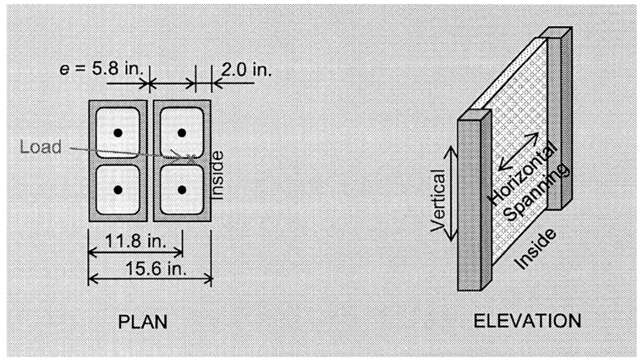
o.c. spacing = 16 ft

f'm = 2000 psi

Gr 60 reinforcement

reinforcement not laterally tied

eccentricity = 5.8 in.



D = 9.6 k W = 8.1 k (uplift) W = 26 psf (lateral)   Load Case: 0.9 D + 1.0 W

The maximum moment will occur approximately at the midheight of the pilaster.

$$M_u = \frac{w_u h^2}{8} + \frac{P_{uf} e}{2} = \frac{416\ \text{lb/ft}(24\ \text{ft})^2}{8} + \frac{(540\ \text{lb})(5.8/12\ \text{ft})}{2} = 30{,}100\ \text{lb}-\text{ft} = 361{,}000\ \text{lb}-\text{in.}$$

Combined Bending and Axial Load

CMU Pilaster Design

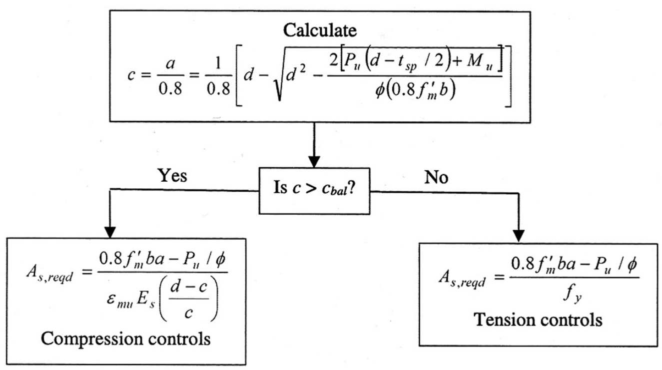


Figure 12.4-4 Flow Chart for Strength Design of Flexure and Axially Loaded Members

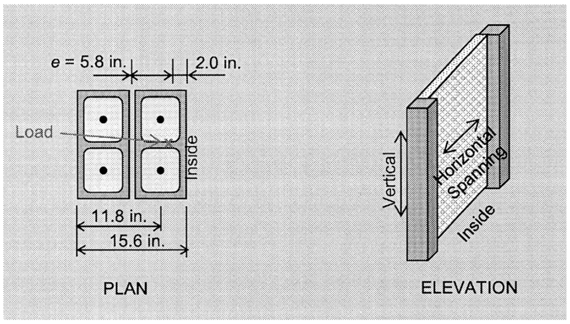
Estimate required area of steel, $A_{s,reqd}$: Use the flow chart in MDG Figure 12.4-4.

Depth to neutral axis: $$c = \frac{1}{0.8}\left[d - \sqrt{d^2 - \frac{2\left[P_u(d - t_{sp}/2) + M_u\right]}{\phi(0.8 f'_m b)}}\right]$$

$$c = \frac{1}{0.8}\left[11.8\ \text{in.} - \sqrt{(11.8\ \text{in.})^2 - \frac{2\left[2{,}700\ \text{lb}(11.8\ \text{in.} - (15.6\ \text{in.})/2) + 361{,}000\ \text{lb - in.}\right]}{0.9(0.8)(2{,}000\ \text{psi})(15.6\ \text{in.})}}\right] = 1.87\ \text{in.}$$

a = 1.496 in.

Combined Bending and Axial Load CMU Pilaster Design



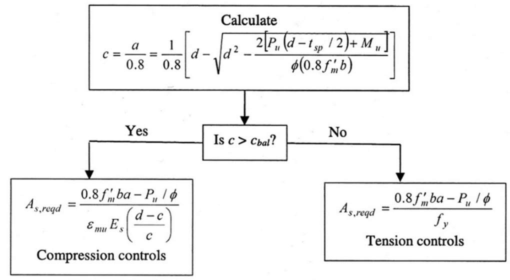


Figure 12.4-4 Flow Chart for Strength Design of Flexure and Axially Loaded Members

Balanced c: $c_{bal} = 0.547d = 0.547(11.8\text{ in.}) = 6.45\text{ in.}$

Since $c \le c_{bal}$, tension controls.

Depth of stress block: $a = 0.8c = 0.8(1.87\text{ in.}) = 1.50\text{ in.}$

Required area of reinforcement: $A_{s,reqd} = \frac{0.8 f'_m ba - P_u/\phi}{f_y}$

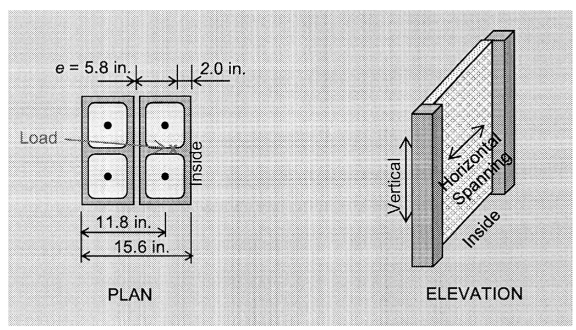
$$A_{s,reqd} = \frac{0.8(2{,}000\text{ psi})(15.6\text{ in.})(1.50\text{ in.}) - 2{,}700\text{ lb}/0.9}{60{,}000\text{ psi}} = 0.574\text{ in.}^2$$

Combined Bending and Axial Load CMU Pilaster Design

$A_{sreqd} = 0.574\text{ in}^2$

try 2 x #5 bars, As = 0.62 in^2

Since wind can be suction or pressure,
bars are placed on both sides, symmetrically,
in the center of each cell – total of 4 #5 bars.



Pu = 2,700 lbs

Mu = 361,000 in-lbs