Architecture 504
Masonry Structures

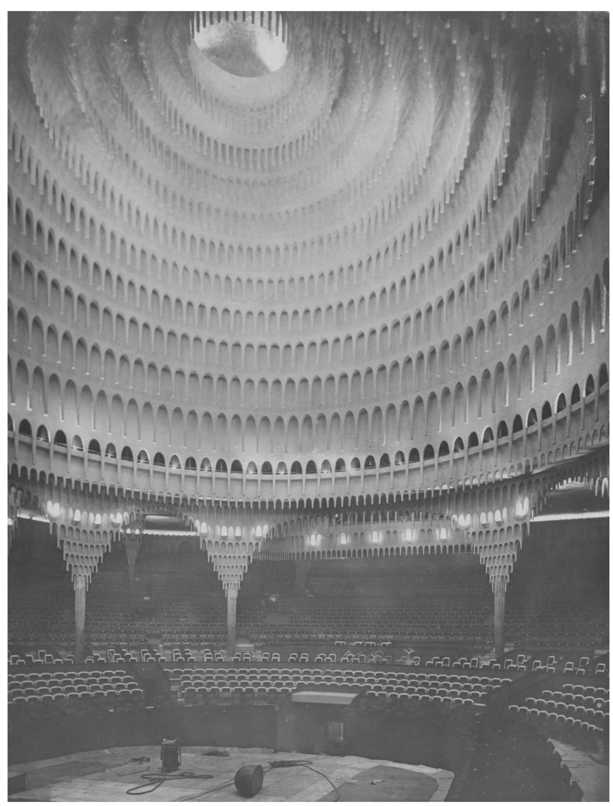
Combined Axial and Flexure Load - pt1.

- Concentric Axial
- Flexure Loading
- Interaction

Hans Poelzig
Großes Schauspielhaus
Berlin, 1918-19

Hans Poelzig’s Berlin Theater

Großes Schauspielhaus
Berlin, 1918-19



Hans Poelzig's Berlin Theater

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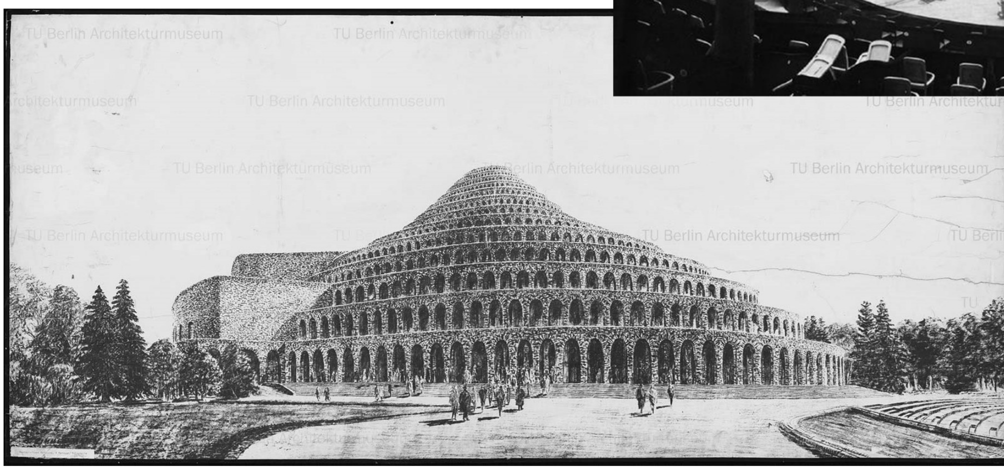


Saltzburg Theater

Concentric Axial Compression

9.3.4.1 Nominal strength

9.3.4.1.1 Nominal axial and flexural strength

Section 4.3.3 Radius of gyration

use r_{avg} because it relates to stiffness

$$P_n = 0.80\left[0.80 f'_m (A_n - A_{st}) + f_y A_{st}\right]\left[1 - \left(\frac{h}{140r}\right)^2\right] \quad \text{for } \frac{h}{r} \le 99$$ eq. 9-15

$$P_n = 0.80[0.80 f'_m (A_n - A_{st}) + f_y A_{st}]\left(\frac{70r}{h}\right)^2 \quad \text{for } \frac{h}{r} > 99$$ eq. 9-16

$\phi = 0.9$ A_{st} = area of laterally tied steel

similar to Euler Equation ($E_m = 900 f'_m$) for CMU

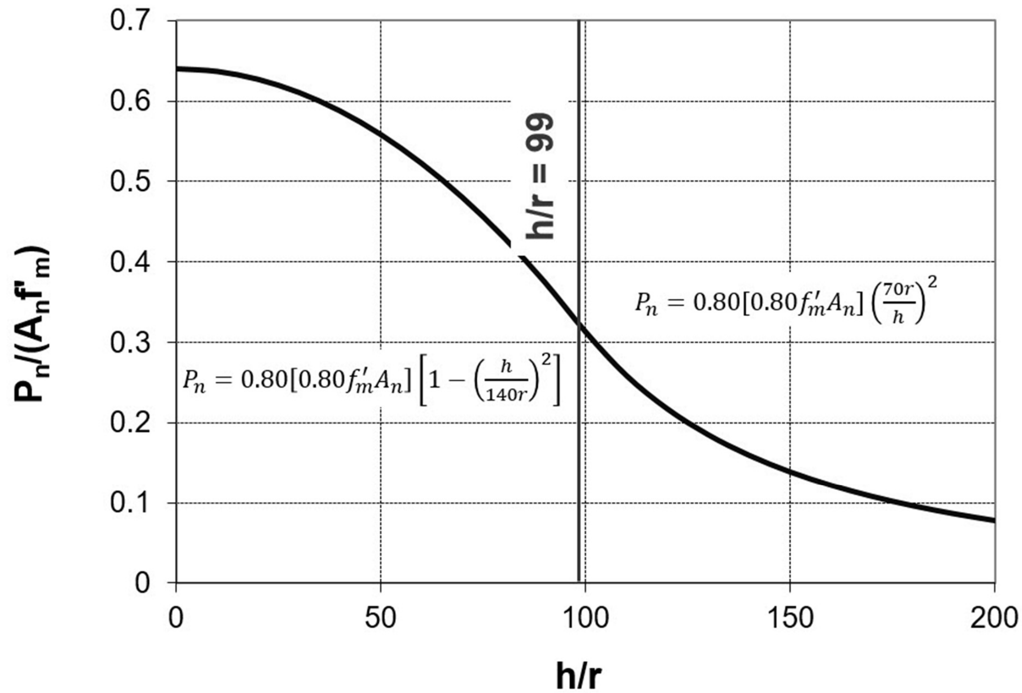
$$P_{euler} = \frac{\pi^2 EI}{h^2} = \frac{\pi^2 E A_n r^2}{h^2} = \frac{\pi^2 (900 f'_m) A_n r^2}{h^2} = A_n f'_m \left(94.2\frac{r}{h}\right)^2$$

Euler Equation above is for CMU with $E_m = 900f'm$

for clay ($E_m = 700 f'_m$), the term is $(83.1r/h)^2$

Concentric Axial Compression

Buckling curve (similar to Euler)



Concentric Axial Compression

Radius of Gyration

TMS 402 CODE

4.3.3 *Radius of gyration*
Radius of gyration shall be calculated using the average net cross-sectional area of the member considered.

COMMENTARY

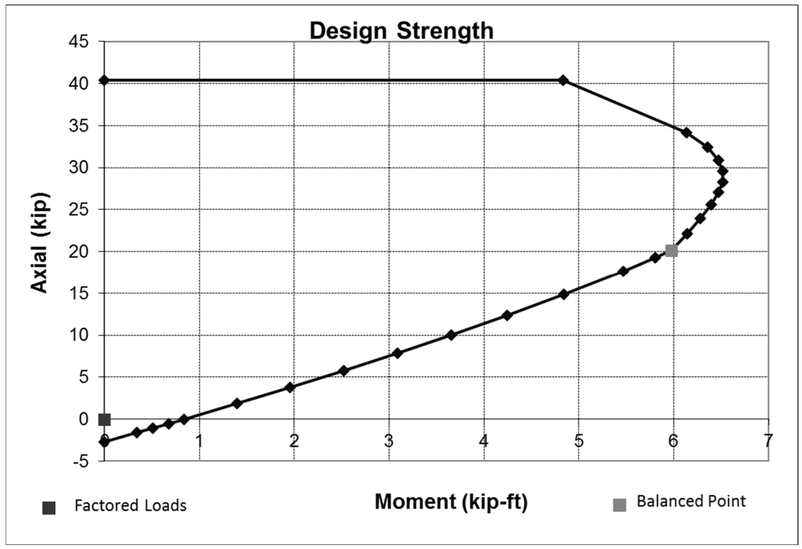
4.3.3 *Radius of gyration*
The radius of gyration is the square root of the ratio of bending moment of inertia to cross-sectional area. Because stiffness is based on the average net cross-sectional area of the member considered, this same area should be used in the calculation of radius of gyration.

see NCMA TEK 14-01B for tabulated values

$$r_{avg} = \sqrt{I_{avg}/A_{avg}}$$

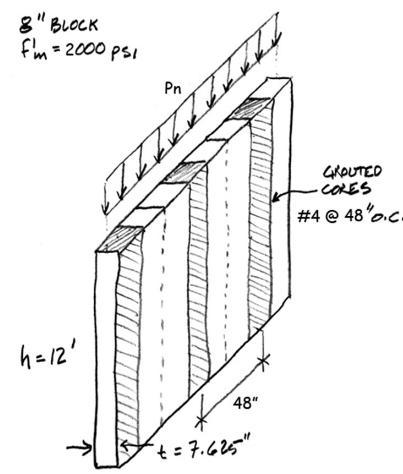
Interaction Diagram

- Assume strain/stress distribution
- Compute forces in masonry and steel
- Sum forces to get axial force
- Sum moment about centerline to get bending moment
- Key points
    - Pure axial load
    - Pure bending
    - Balanced



Concentric Axial Compression

Example – 8 in. CMU bearing wall f'm = 2000 psi
- Find capacity



Pure Axial Compression – no Moment
Steel not tied by 5.3.1.4

NCMA TEK 14-1B Section Properties of Concrete Masonry Walls

r_{avg} = 2.66 in. A_n = 40.7in.^{2}/ft I_n = 332.0 in.^{4}/ft
A_{st} = 0.2in^2/4ft = 0.05 in^2/ft
Find h/r = 144"/2.66" = 54.1 < 99

3a: Horizontal Section Properties (Masonry Spanning Vertically)

			Net cross-sectional properties[A]			Average cross-sectional properties[B]			
Unit	Grout spacing (in.)	Mortar bedding	A_n (in.^{2}/ft)	I_n (in.^{4}/ft)	S_n (in.^{3}/ft)	A_{avg} (in.^{2}/ft)	I_{avg} (in.^{4}/ft)	S_{avg} (in.^{3}/ft)	r_{avg} (in.)
Hollow	No grout	Face shell	30.0	308.7	81.0	41.5	334.0	87.6	2.84
Hollow	No grout	Full	41.5	334.0	87.6	41.5	334.0	87.6	2.84
100% solid/solidly grouted		Full	91.5	443.3	116.3	91.5	443.3	116.3	2.20
Hollow	16	Face shell	62.0	378.6	99.3	65.8	387.1	101.5	2.43
Hollow	24	Face shell	51.3	355.3	93.2	57.7	369.4	96.9	2.53
Hollow	32	Face shell	46.0	343.7	90.1	53.7	360.5	94.6	2.59
Hollow	40	Face shell	42.8	336.7	88.3	51.2	355.2	93.2	2.63
Hollow	48	Face shell	40.7	332.0	87.1	49.6	351.7	92.2	2.66
Hollow	72	Face shell	37.1	324.3	85.0	46.9	345.8	90.7	2.71
Hollow	96	Face shell	35.3	320.4	84.0	45.6	342.8	89.9	2.74
Hollow	120	Face shell	34.3	318.0	83.4	44.8	341.0	89.5	2.76

TMS 402 Chapter 9

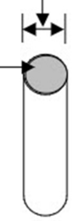
9.1.4 Strength-reduction factors, ø

Action	Reinforced Masonry	Unreinforced Masonry
combinations of flexure and axial load	0.90	0.60
shear	0.80	
bearing	0.60	
anchor bolts: pryout	0.50	
anchor bolts: controlled by anchor bolt steel	0.90	
anchor bolts: pullout	0.65	

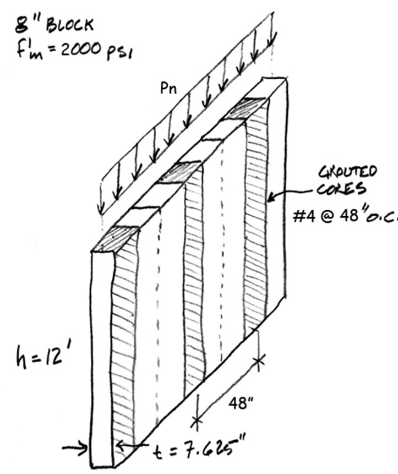
Bar Dia (inch) = Bar No. / 8

Table-1: Diameter and Area of Reinforcing Bars

Bar No.	Bar Dia. (in)	Area (Sq.in)
#3	3/8	0.11
#4	1/2	0.20
#5	5/8	0.31
#6	3/4	0.44
#7	7/8	0.60
#8	1	0.79
#9	9/8	1.00
#10	10/8	1.27
#11	11/8	1.56



Concentric Axial Compression



Example – 8 in. CMU bearing wall f'm = 2000 psi
- Find capacity

Pure Axial Compression – no Moment

Steel not tied by 5.3.1.4

NCMA TEK 14-1B Section Properties of Concrete Masonry Walls

r = 2.66 in. A_n = 40.7in.^{2}/ft I_n = 332.0 in.^{4}/ft

A_{st} = 0.05 in^2/ft

Find h/r = 144"/2.66" = 54.1 < 99

$$P_n = 0.80\left[0.80 f'_m (A_n - A_{st}) + f_y A_{st}\right]\left[1 - \left(\frac{h}{140r}\right)^2\right]$$

Nominal Axial Load, P_n

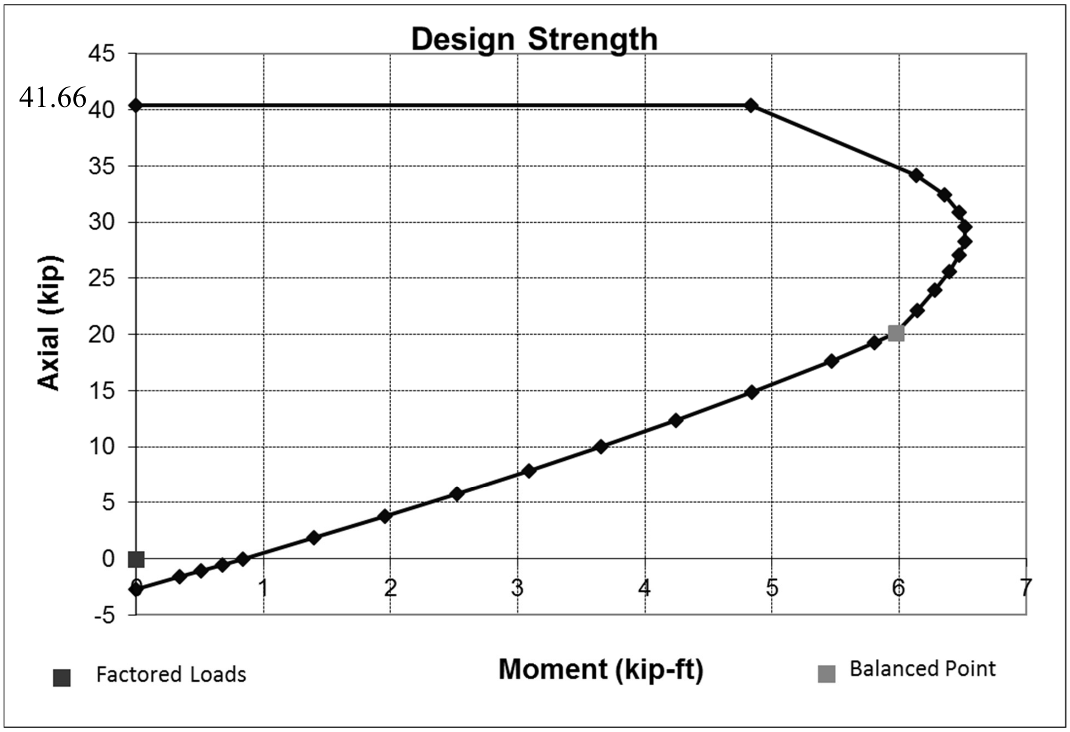
$$= 0.80\left[0.80(2.0ksi)\left(40.7\frac{in.^2}{ft} - 0.05\right) + 60(0.05)\right]\left[1 - \left(\frac{144}{372.4}\right)^2\right]$$

$$= 54.43\frac{k}{ft}(0.85) = 46.29\frac{k}{ft}$$

Design axial load, ϕP_n

$$\phi P_n = 0.9\left(46.29\frac{k}{ft}\right) = 41.66\frac{k}{ft}$$

Interaction Diagram

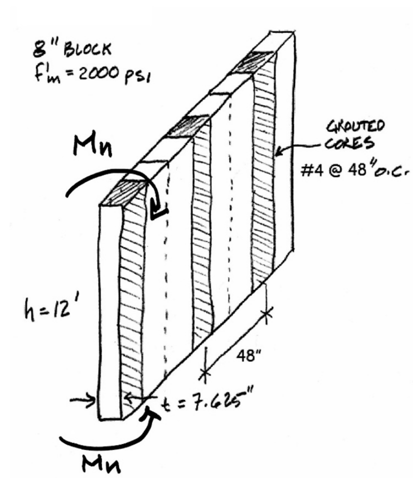


Moment only (no axial load)

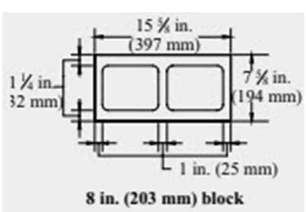
Example – 8 in. CMU bearing wall - Find capacity

NCMA TEK 14-1B Section Properties of Concrete Masonry Walls

r = 2.66 in. A_n = 40.7in.^{2}/ft I_n = 332.0 in.^{4}/ft



A_s for #4



$$A_s = \frac{0.20in.^2}{48in.}\frac{12in.}{ft} = 0.05\frac{in.^2}{ft}$$

$$a = \frac{A_s f_y}{0.8bf'_m} = \frac{0.05\frac{in.^2}{ft}(60ksi)}{0.8\left(12\frac{in.}{ft}\right)(2.0ksi)} = 0.156in.$$

Check to make sure stress block is in face shell

Nominal moment, M_n

$$M_n = A_s f_y\left(d - \frac{a}{2}\right)$$

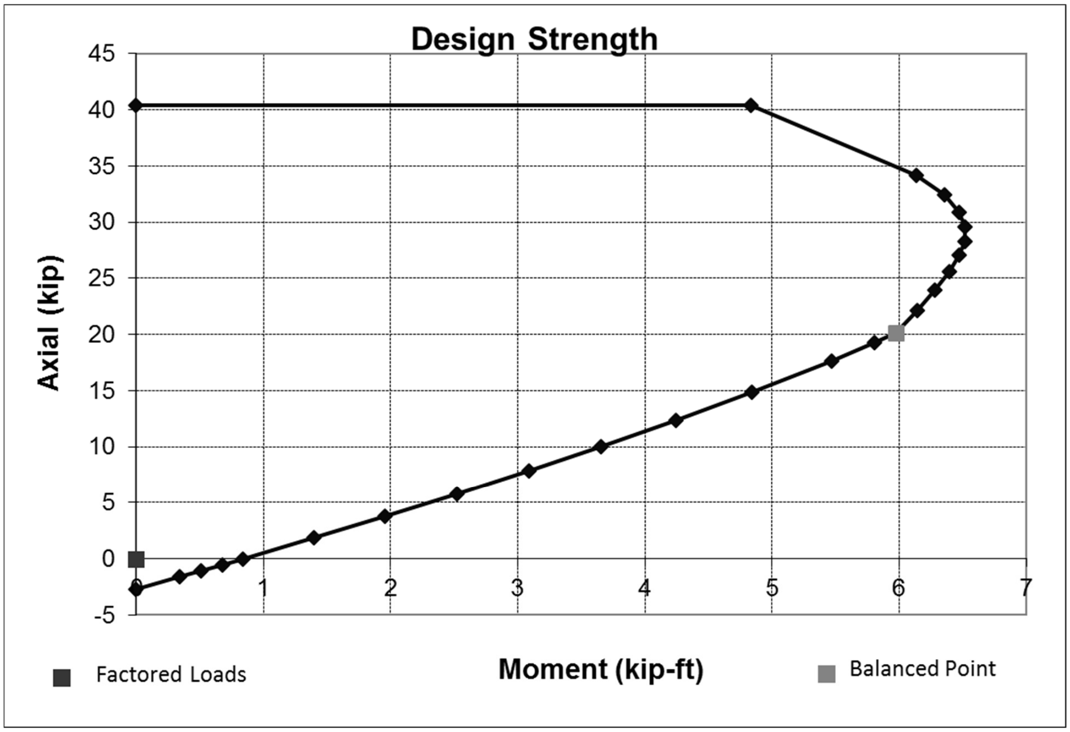
$$= 0.05\frac{in.^2}{ft}(60ksi)\left(3.81in. - \frac{0.156in}{2}\right)$$

$$= 3.0(3.73) = 11.19\frac{k\cdot in}{ft} = 0.934\frac{k\cdot ft}{ft}$$

Design moment, ϕM_n

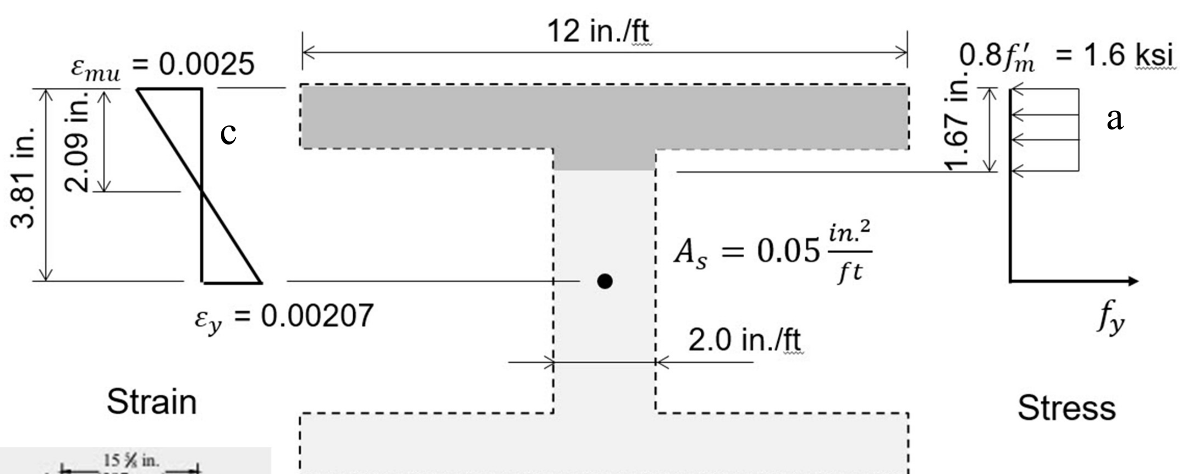
$$\phi M_n = 0.9\left(0.924\frac{k\cdot ft}{ft}\right) = 0.840\frac{k\cdot ft}{ft}$$

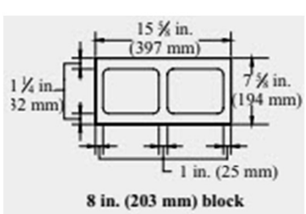
Interaction Diagram



Balanced

Example – 8 in. CMU bearing wall



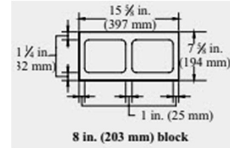


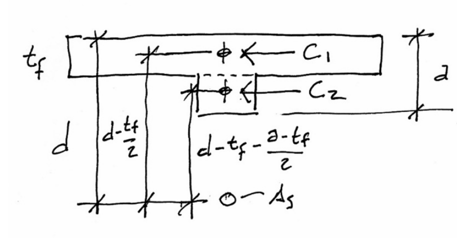
web length = $\frac{8in.}{48in.}\frac{12in.}{ft} = 2.0\,\frac{in.}{ft}$

$$c = \frac{\varepsilon_{mu}}{\varepsilon_{mu} + \varepsilon_y} d = \frac{0.0025}{0.0025 + 0.00207} 3.81in. = 2.09in.$$

$$a = 0.8c = 0.8(2.09in.) = 1.67in.$$

Balanced Example – 8 in. CMU bearing wall





Compressive force, C_m

$$C_{m,face\ shell} = C_1 = [0.8(f'm\ ksi)](t_f\ in.)\left(12\frac{in.}{ft}\right)$$

$$C_{m,face\ shell} = [0.8(2.0ksi)](1.25in.)\left(12\frac{in.}{ft}\right) = 24\frac{k}{ft}$$

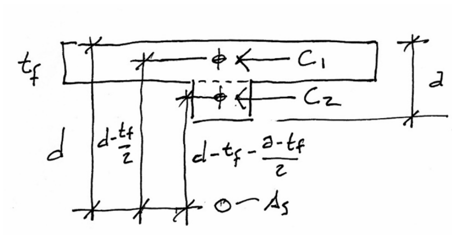
$$C_{m,web} = C_2 = [0.8(f'm\ ksi)](a\ in. - t_f in.)\left(b_w\frac{in.}{ft}\right)$$

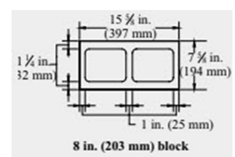
$$C_{m,web} = [0.8(2.0ksi)](1.67in. - 1.25in.)\left(2.0\frac{in.}{ft}\right) = 1.34\frac{k}{ft}$$

Tension force, T

$$T = f_y A_s = 60ksi\left(0.05\frac{in.^2}{ft}\right) = 3.0\frac{k}{ft}$$

Balanced Example – 8 in. CMU bearing wall





Design force, ϕP_n

$$\phi P_n = 0.9(C_1 + C_2 - T)$$

$$\phi P_n = 0.9(24.0 + 1.34 - 3.0)\frac{k}{ft} = 20.1\frac{k}{ft}$$

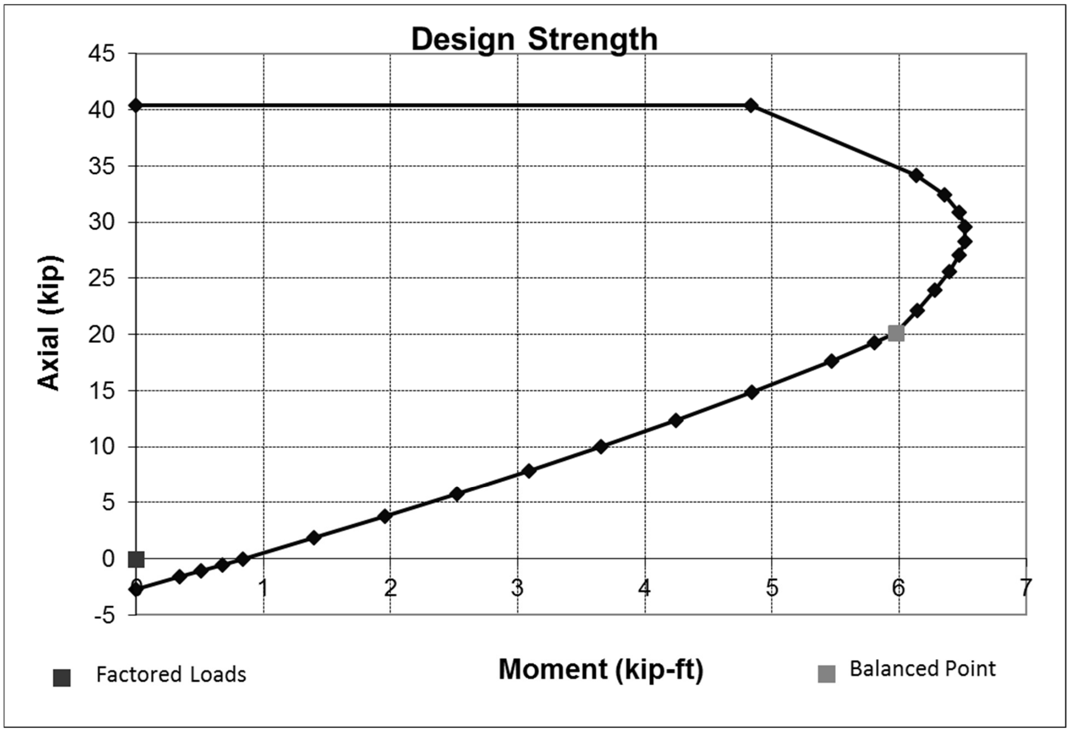
Design moment, ϕM_n

$$\phi M_n = 0.9\left[C_1\left(d - \frac{t_f}{2}\right) + C_2\left(d - tf - \frac{a - t_f}{2}\right)\right]$$

$$\phi M_n = 0.9\left[24.0\frac{k}{ft}\left(3.81 - \frac{1.25}{2}\right)in./(12) + 1.34\frac{k}{ft}\left(3.81 - 1.25 - \frac{1.67 - 1.25}{2}\right)in./(12)\right] = 5.97\frac{k \cdot ft}{ft}$$

$$\phi P_n = 20.1\frac{k}{ft} \qquad \phi M_n = 5.97\frac{k \cdot ft}{ft}$$

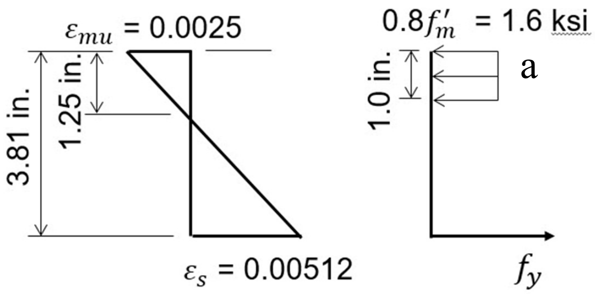
Interaction Diagram



Below Balanced

Example – 8 in. CMU bearing wall

Below Balanced:
c = 1.25 in.



$$\phi P_n = 14.6\,\frac{k}{ft}$$
$$\phi M_n = 4.76\,\frac{k\cdot ft}{ft}$$

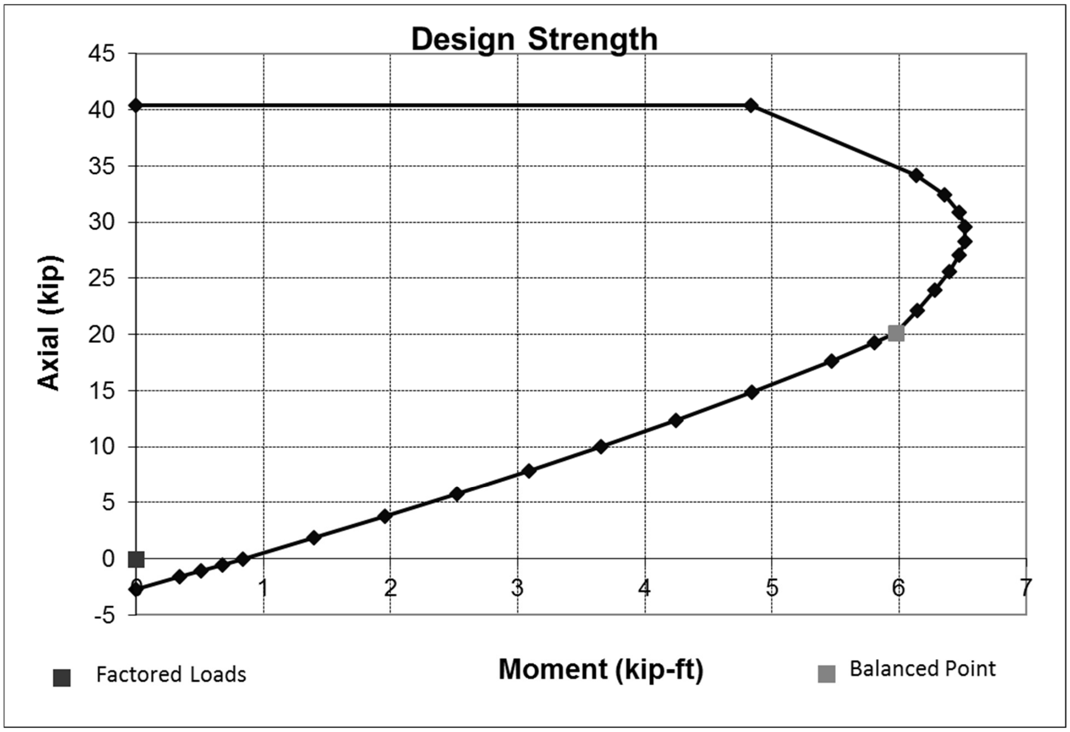
$$C_m = [0.8(2.0ksi)](1.0in.)\left(12\,\frac{in.}{ft}\right) = 19.2\,\frac{k}{ft}$$

$$T = f_y A_s = 60ksi\left(0.05\,\frac{in.^2}{ft}\right) = 3.0\,\frac{k}{ft}$$

$$\phi P_n = 0.9(19.2 - 3.0)\,\frac{k}{ft} = 14.6\,\frac{k}{ft}$$

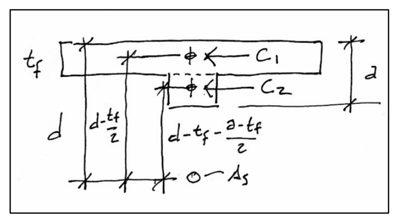
$$\phi M_n = 0.9\left[19.2\,\frac{k}{ft}\left(3.81 - \frac{1.0}{2}\right) in./(12)\right] = 4.76\,\frac{k\cdot ft}{ft}$$

Interaction Diagram

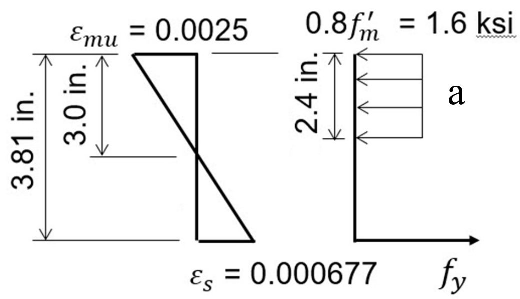


Above Balanced

Example – 8 in. CMU bearing wall



Above Balanced:
c = 3.0 in.



$$\phi P_n = 24.0\frac{k}{ft}$$
$$\phi M_n = 6.28\frac{k\cdot ft}{ft}$$

$$C_{m,fs} = [0.8(2.0ksi)](1.25in.)\left(12\frac{in.}{ft}\right) = 24.0\frac{k}{ft}$$
$$C_{m,web} = [0.8(2.0ksi)](2.4in. - 1.25in.)\left(2.0\frac{in.}{ft}\right) = 3.68\frac{k}{ft}$$

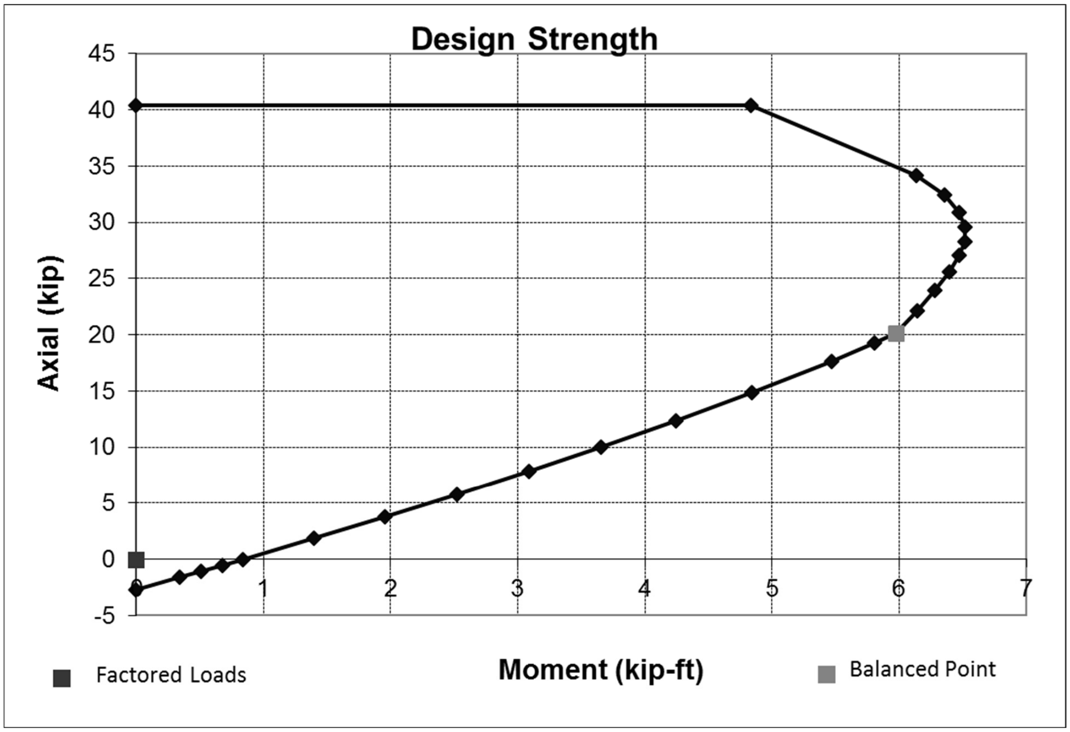
$$T = E_s\varepsilon_s A_s = 29000ksi(0.000677)\left(0.05\frac{in.^2}{ft}\right) = 0.98\frac{k}{ft}$$

$$\phi P_n = 0.9(24.0 + 3.68 - 0.98)\frac{k}{ft} = 24.0\frac{k}{ft}$$

$$\phi M_n = 0.9\left[24.0\frac{k}{ft}\left(3.81 - \frac{1.25}{2}\right)in. + 3.68\frac{k}{ft}\left(3.81 - 1.25 - \frac{2.4-1.25}{2}\right)in.\right]$$
$$= 6.28\frac{k\cdot ft}{ft}$$

Interaction Diagram

Example – 8 in. CMU bearing wall



Interaction Diagram

Example – 8 in. CMU bearing wall

Point	c (in.)	$C_{m,fs}$ (kip/ft)	$C_{m,web}$ (kip/ft)	T (kip/ft)	ϕP_n (kip/ft)	ϕM_n (kip-ft/ft)
$a = d$	4.76	24.0	8.2	0	29.0	6.52
$c = d$	3.81	24.0	5.8	0	26.8	6.45
	3.00	24.0	3.7	1.0	24.0	6.28
Balanced	2.09	24.0	1.3	3.0	20.1	5.97
a = 1.25 in.	1.56	24.0	0	3.0	18.9	5.73
	1.25	19.2	0	3.0	14.6	4.77
	1.0	15.4	0	3.0	11.1	3.93
	0.8	12.3	0	3.0	8.4	3.22
	0.6	9.2	0	3.0	5.6	2.47
	0.4	6.1	0	3.0	2.8	1.68
Pure Moment	0.195	3.0	0	3.0	0	0.84

Interaction Diagram

Solid vs. Partial Grout