Architecture 504
Masonry Structures

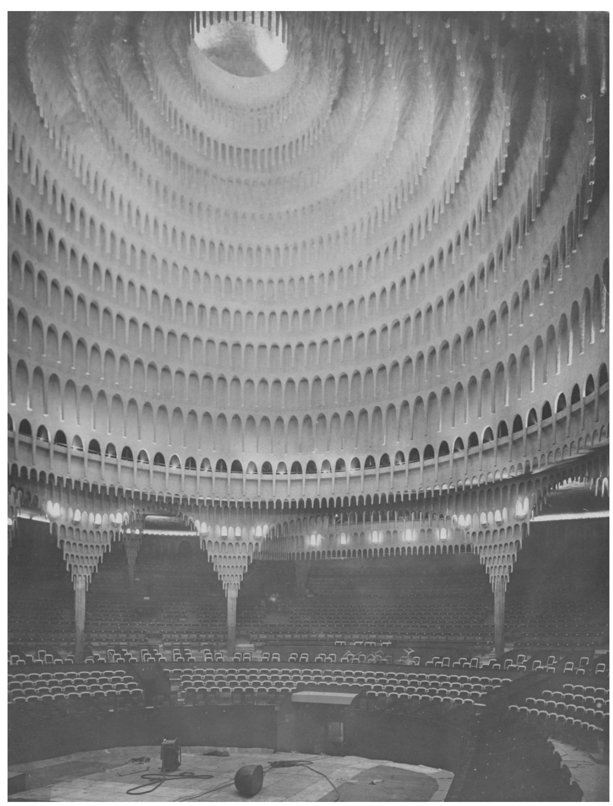
Combined Axial and Flexure Load

- Concentric axial
- Interaction
- Bearing walls

Hans Poelzig
Großes Schauspielhaus
Berlin, 1918-19

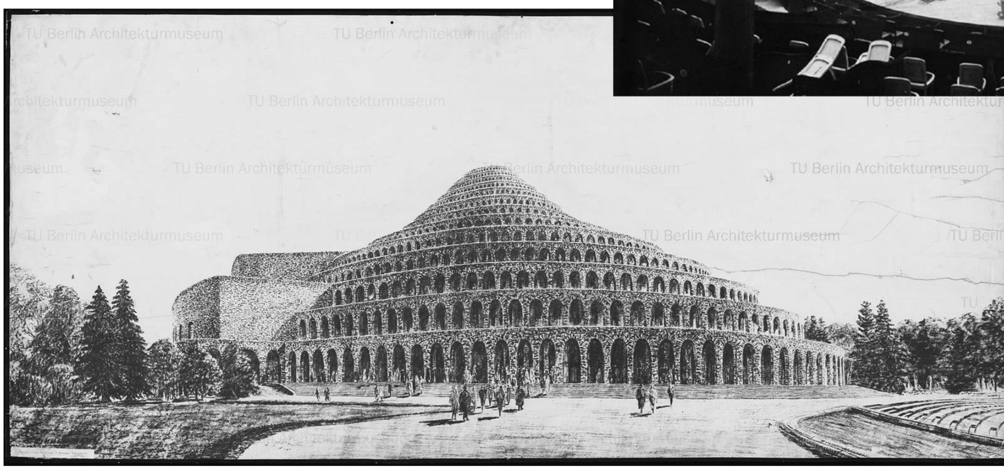
Hans Poelzig’s Berlin Theater

Großes Schauspielhaus
Berlin, 1918-19



Hans Poelzig's Berlin Theater

Großes Schauspielhaus
Berlin, 1918-19



Saltzburg Theater

Concentric Axial Compression

9.3.4.1 Nominal strength
- 9.3.4.1.1 Nominal axial and flexural strength
  - Section 4.3.3 Radius of gyration
  - use r_{avg} because it relates to stiffness

$$P_n = 0.80\left[0.80 f'_m (A_n - A_{st}) + f_y A_{st}\right]\left[1 - \left(\frac{h}{140r}\right)^2\right] \quad \text{for } \frac{h}{r} \le 99$$ eq. 9-15

$$P_n = 0.80\left[0.80 f'_m (A_n - A_{st}) + f_y A_{st}\right]\left(\frac{70r}{h}\right)^2 \quad \text{for } \frac{h}{r} > 99$$ eq. 9-16

$\phi = 0.9$ $\quad A_{st}$ = area of **laterally tied steel**

similar to Euler Equation:

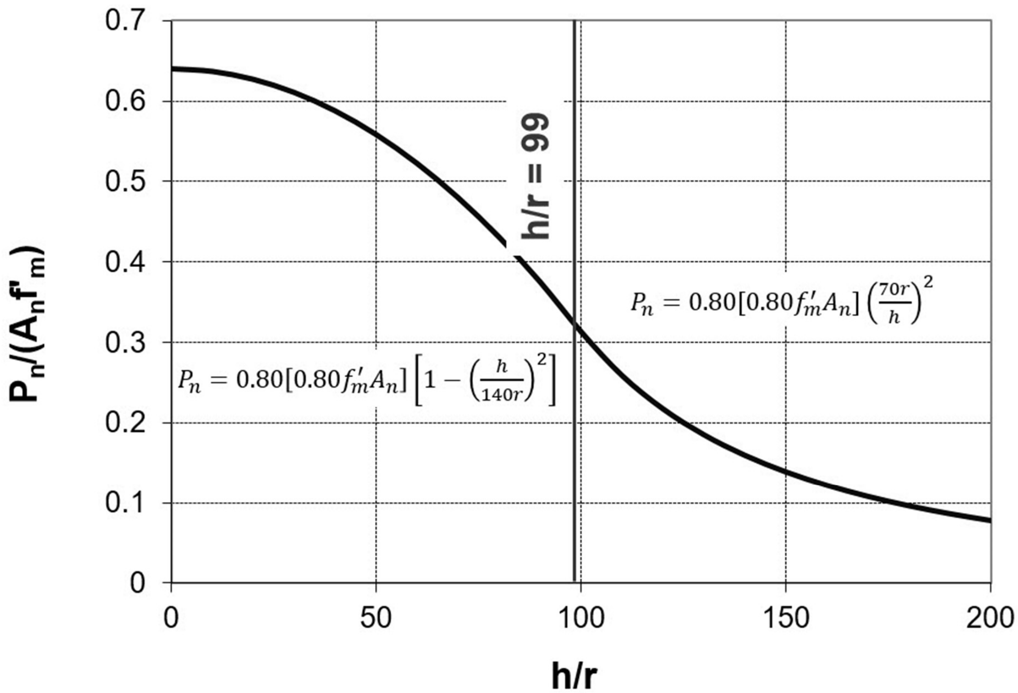
$$P_{euler} = \frac{\pi^2 EI}{h^2} = \frac{\pi^2 E A_n r^2}{h^2} = \frac{\pi^2 (900 f'_m) A_n r^2}{h^2} = A_n f'_m \left(94.2\frac{r}{h}\right)^2$$

Euler Equation above is for CMU

for clay $(E_m = 700 f'_m)$, the term is $(83.1r/h)^2$

Concentric Axial Compression

Buckling curve (similar to Euler)



Concentric Axial Compression

A_{st} = area of **laterally tied steel**

TMS 402 CODE

5.3.1.4 *Lateral ties* — Lateral ties shall conform to the following:

(a) Vertical reinforcement shall be enclosed by lateral ties at least 1/4 in. (6.4 mm) in diameter.

(b) Vertical spacing of lateral ties shall not exceed 16 longitudinal bar diameters, 48 lateral tie bar or wire diameters, or least cross-sectional dimension of the member.

(c) Lateral ties shall be arranged so that every corner and alternate longitudinal bar shall have lateral support provided by the corner of a lateral tie with an included angle of not more than 135 degrees. No bar shall be farther than 6 in. (152 mm) clear on each side along the lateral tie from such a laterally supported bar. Lateral ties shall be placed in grout. Where longitudinal bars are located around the perimeter of a circle, a complete circular lateral tie is permitted. Lap length for circular ties shall be 48 tie diameters.

(d) Lateral ties shall be located vertically not more than one-half lateral tie spacing above the top of footing or slab in any story, and shall be spaced not more than one-half a lateral tie spacing below the lowest horizontal reinforcement in beam, girder, slab, or drop panel above.

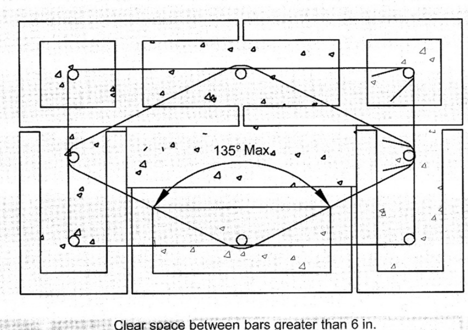


Figure CC-5.3-2 — Example of a lateral tie included angle for a CMU column

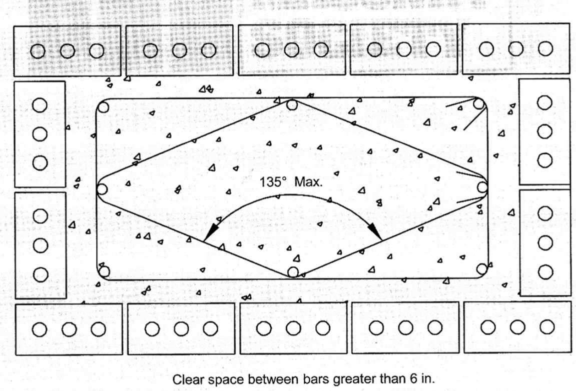


Figure CC-5.3-3 — Example of a lateral tie included angle for a clay masonry column

Concentric Axial Compression

Radius of Gyration

TMS 402 CODE

4.3.3 *Radius of gyration*
Radius of gyration shall be calculated using the average net cross-sectional area of the member considered.

COMMENTARY

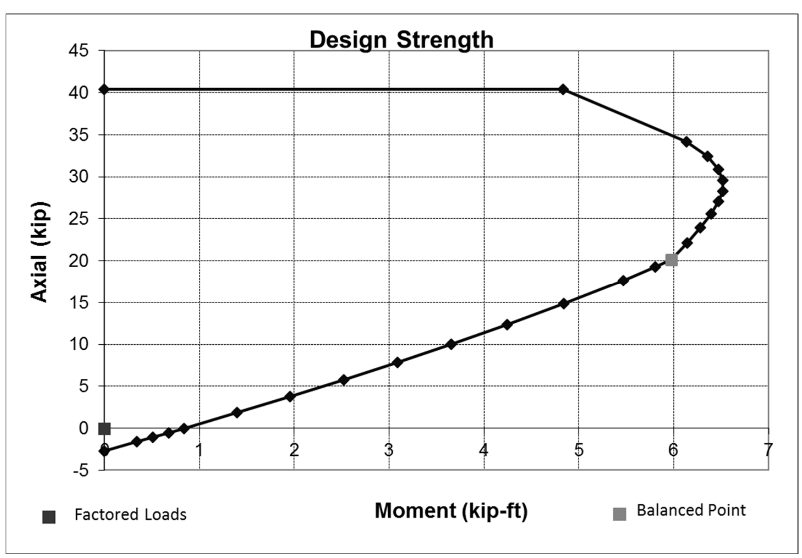
4.3.3 *Radius of gyration*
The radius of gyration is the square root of the ratio of bending moment of inertia to cross-sectional area. Because stiffness is based on the average net cross-sectional area of the member considered, this same area should be used in the calculation of radius of gyration.

see NCMA TEK 14-01B for tabulated values

$$r = \sqrt{I/A}$$

Interaction Diagram

- Assume strain/stress distribution
- Compute forces in masonry and steel
- Sum forces to get axial force
- Sum moment about centerline to get bending moment
- Key points
  - Pure axial load
  - Pure bending
  - Balanced



Concentric Axial Compression

Example – 8 in. CMU bearing wall   f'm = 2000 psi
- Find capacity

Pure Axial Compression – no Moment

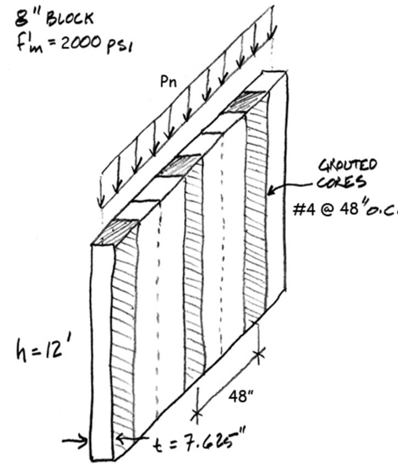
Steel not tied by 5.3.1.4

NCMA TEK 14-1B Section Properties of Concrete Masonry Walls

r = 2.66 in. A_n = 40.7in.^{2}/ft I_n = 332.0 in.^{4}/ft

A_{st} = 0.05 in^2/ft

Find h/r = 144"/2.66" = 54.1 < 99



3a: Horizontal Section Properties (Masonry Spanning Vertically)										
	Grout	Mortar	Net cross-sectional properties[A]			Average cross-sectional properties[B]				
Unit	spacing (in.)	bedding	A_n (in.^{2}/ft)	I_n (in.^{4}/ft)	S_n (in.^{3}/ft)	A_{avg} (in.^{2}/ft)	I_{avg} (in.^{4}/ft)	S_{avg} (in.^{3}/ft)	r_{avg} (in.)
Hollow	No grout	Face shell	30.0	308.7	81.0	41.5	334.0	87.6	2.84
Hollow	No grout	Full	41.5	334.0	87.6	41.5	334.0	87.6	2.84
100% solid/solidly grouted		Full	91.5	443.3	116.3	91.5	443.3	116.3	2.20
Hollow	16	Face shell	62.0	378.6	99.3	65.8	387.1	101.5	2.43
Hollow	24	Face shell	51.3	355.3	93.2	57.7	369.4	96.9	2.53
Hollow	32	Face shell	46.0	343.7	90.1	53.7	360.5	94.6	2.59
Hollow	40	Face shell	42.8	336.7	88.3	51.2	355.2	93.2	2.63
Hollow	48	Face shell	40.7	332.0	87.1	49.6	351.7	92.2	2.66
Hollow	72	Face shell	37.1	324.3	85.0	46.9	345.8	90.7	2.71
Hollow	96	Face shell	35.3	320.4	84.0	45.6	342.8	89.9	2.74
Hollow	120	Face shell	34.3	318.0	83.4	44.8	341.0	89.5	2.76

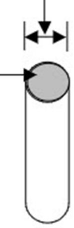
TMS 402 Chapter 9

9.1.4 Strength-reduction factors, ø

Action	Reinforced Masonry	Unreinforced Masonry
combinations of flexure and axial load	0.90	0.60
shear	0.80	
bearing	0.60	
anchor bolts: pryout	0.50	
anchor bolts: controlled by anchor bolt steel	0.90	
anchor bolts: pullout	0.65	

Bar Dia (inch) = Bar No. / 8

Table-1: Diameter and Area of Reinforcing Bars		
Bar No.	Bar Dia. (in)	Area (Sq.in)
#3	3/8	0.11
#4	1/2	0.20
#5	5/8	0.31
#6	3/4	0.44
#7	7/8	0.60
#8	1	0.79
#9	9/8	1.00
#10	10/8	1.27
#11	11/8	1.56



Concentric Axial Compression

Example – 8 in. CMU bearing wall   f'm = 2000 psi
 - Find capacity

Pure Axial Compression – no Moment

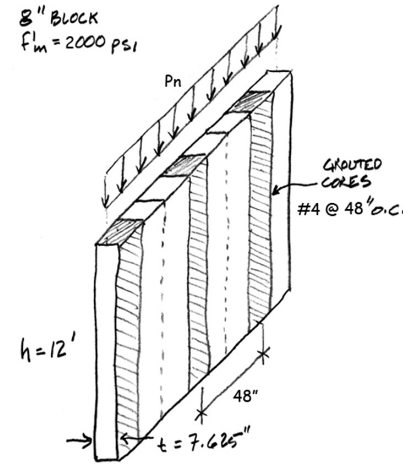
Steel not tied by 5.3.1.4

NCMA TEK 14-1B Section Properties of Concrete Masonry Walls

r = 2.66 in. A_n = 40.7in.^{2}/ft I_n = 332.0 in.^{4}/ft

A_{st} = 0.05 in^2/ft

Find h/r = 144"/2.66" = 54.1 < 99



Nominal Axial Load, P_n

$$P_n = 0.80\left[0.80 f'_m (A_n - A_{st}) + f_y A_{st}\right]\left[1 - \left(\frac{h}{140r}\right)^2\right]$$

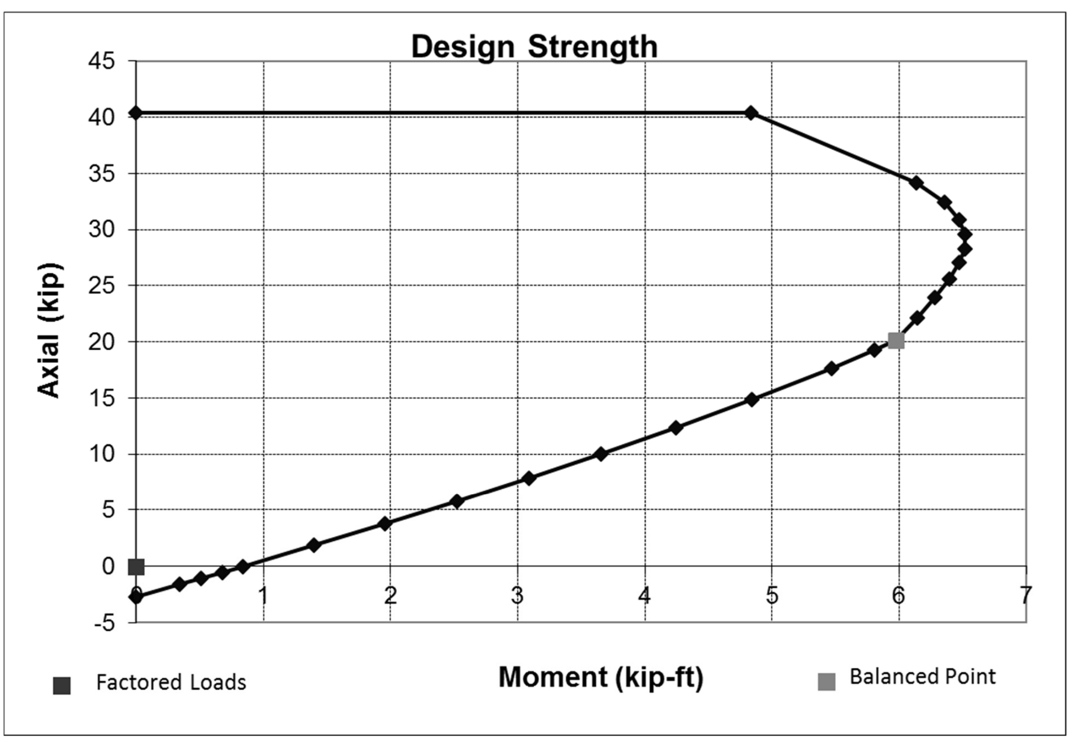
$$= 0.80\left[0.80(2.0ksi)(40.7\frac{in.^2}{ft} - 0.05) + 0.05\right]\left[1 - \left(\frac{54.1}{140}\right)^2\right]$$

$$= 52.1\frac{k}{ft}(0.851) = 44.3\frac{k}{ft}$$

Design axial load, ϕP_n

$$\phi P_n = 0.9\left(44.3\frac{k}{ft}\right) = 39.9\frac{k}{ft}$$

Interaction Diagram

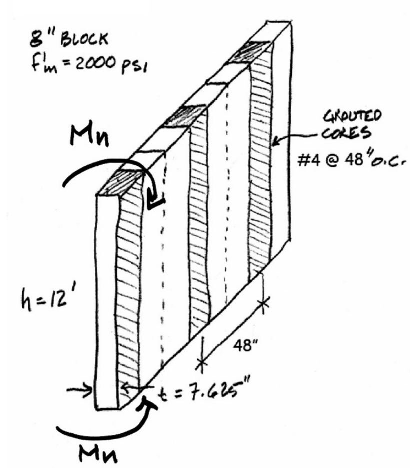


Moment only (no axial load)

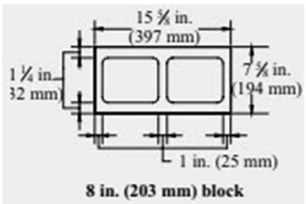
Example – 8 in. CMU bearing wall - Find capacity

NCMA TEK 14-1B Section Properties of Concrete Masonry Walls

r = 2.66 in. A_n = 40.7in.^{2}/ft I_n = 332.0 in.^{4}/ft



A_s for #4



$$A_s = \frac{0.20in.^2}{48in.}\frac{12in.}{ft} = 0.05\frac{in.^2}{ft}$$

$$a = \frac{A_s f_y}{0.8bf'_m} = \frac{0.05\frac{in.^2}{ft}(60ksi)}{0.8\left(12\frac{in.}{ft}\right)(2.0ksi)} = 0.156in.$$

Check to make sure stress block is in face shell

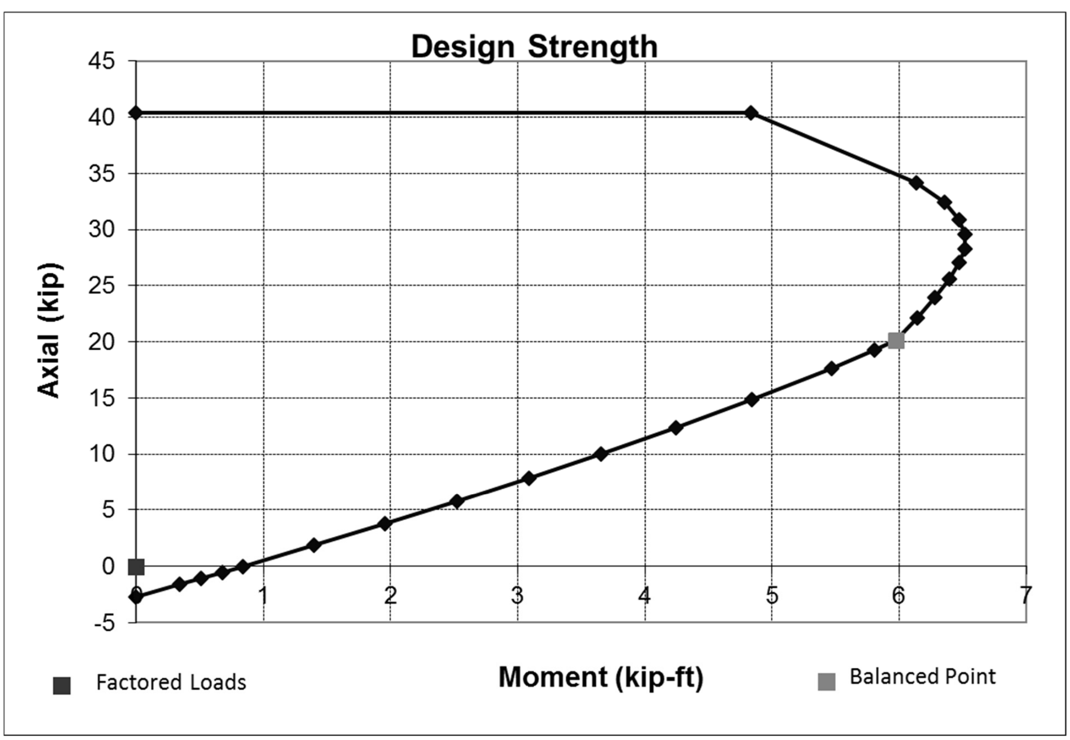
Nominal moment, M_n

$$M_n = A_s f_y\left(d - \frac{a}{2}\right)$$
$$= 0.05\frac{in.^2}{ft}(60ksi)\left(3.81in. - \frac{0.156in}{2}\right)$$
$$= 11.2\frac{k{\cdot}in}{ft} = 0.934\frac{k{\cdot}ft}{ft}$$

Design moment, ϕM_n

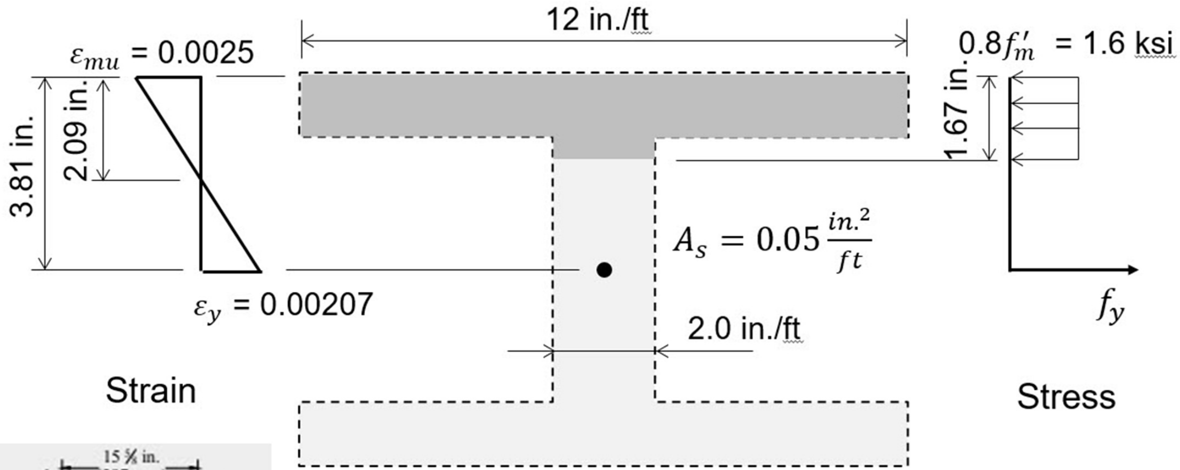
$$\phi M_n = 0.9\left(0.924\frac{k{\cdot}ft}{ft}\right) = 0.840\frac{k{\cdot}ft}{ft}$$

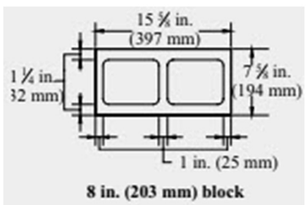
Interaction Diagram



Balanced

Example – 8 in. CMU bearing wall



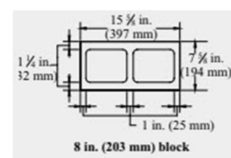


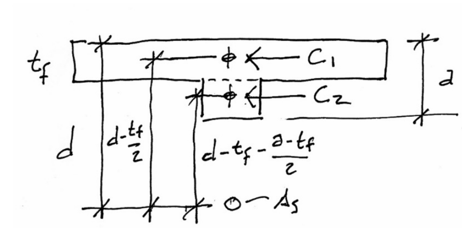
$$\text{web length} = \frac{8in.}{48in.}\frac{12in.}{ft} = 2.0\frac{in.}{ft}$$

$$c = \frac{\varepsilon_{mu}}{\varepsilon_{mu}+\varepsilon_y}d = \frac{0.0025}{0.0025+0.00207}3.81in. = 2.09in.$$

$$a = 0.8c = 0.8(2.09in.) = 1.67in.$$

Balanced Example – 8 in. CMU bearing wall





Compressive force, C_m

$$C_{m,face\ shell} = C_1 = [0.8(f'm\ ksi)](t_f\ in.)\left(12\frac{in.}{ft}\right)$$

$$C_{m,face\ shell} = [0.8(2.0ksi)](1.25in.)\left(12\frac{in.}{ft}\right) = 24\frac{k}{ft}$$

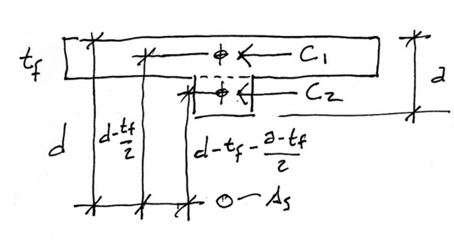
$$C_{m,web} = C_2 = [0.8(f'm\ ksi)](a\ in. - bf in.)\left(b_w\frac{in.}{ft}\right)$$

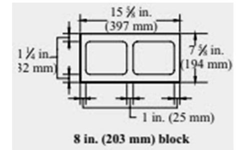
$$C_{m,web} = [0.8(2.0ksi)](1.67in. - 1.25in.)\left(2.0\frac{in.}{ft}\right) = 1.34\frac{k}{ft}$$

Tension force, T

$$T = f_y A_s = 60ksi\left(0.05\frac{in.^2}{ft}\right) = 3.0\frac{k}{ft}$$

Balanced Example – 8 in. CMU bearing wall





Design force, ϕP_n

$$\phi P_n = 0.9(C_1 + C_2 - T)$$

$$\phi P_n = 0.9(24.0 + 1.34 - 3.0)\frac{k}{ft} = 20.1\frac{k}{ft}$$

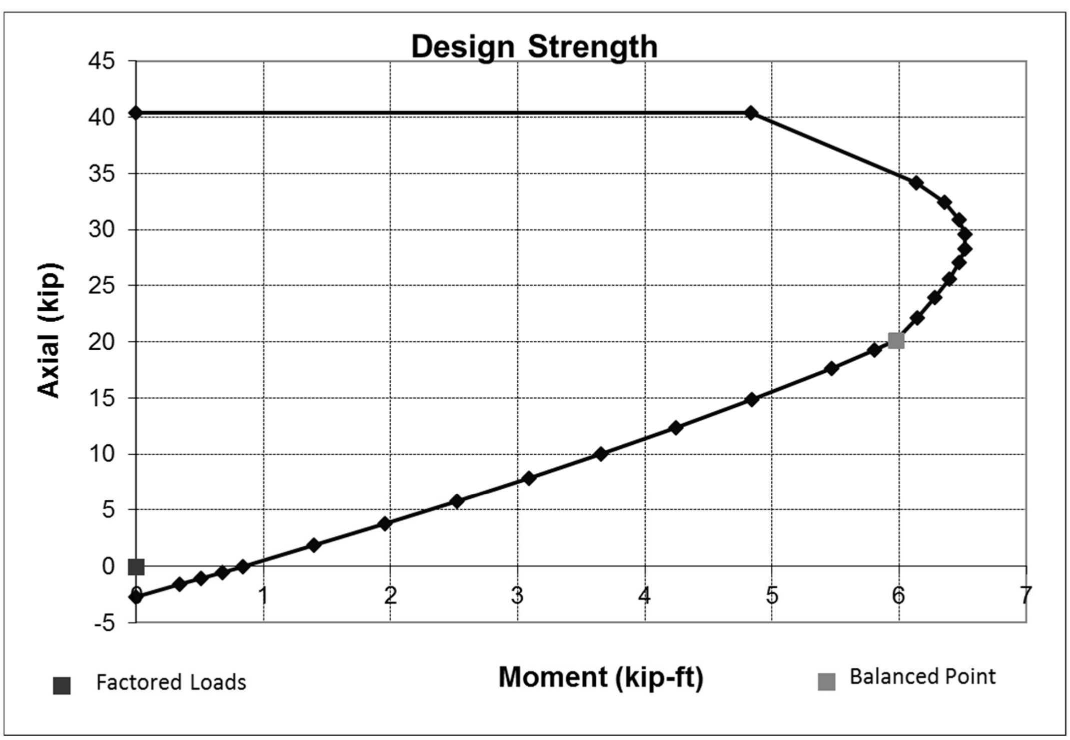
Design moment, ϕM_n

$$\phi M_n = 0.9\left[C_1\left(d - \frac{t_f}{2}\right) + C_2\left(d - t_f - \frac{a - t_f}{2}\right)\right]$$

$$\phi M_n = 0.9\left[24.0\frac{k}{ft}\left(3.81 - \frac{1.25}{2}\right) in./(12) + 1.34\frac{k}{ft}\left(3.81 - 1.25 - \frac{1.67 - 1.25}{2}\right) in./(12)\right] = 5.97\frac{k \cdot ft}{ft}$$

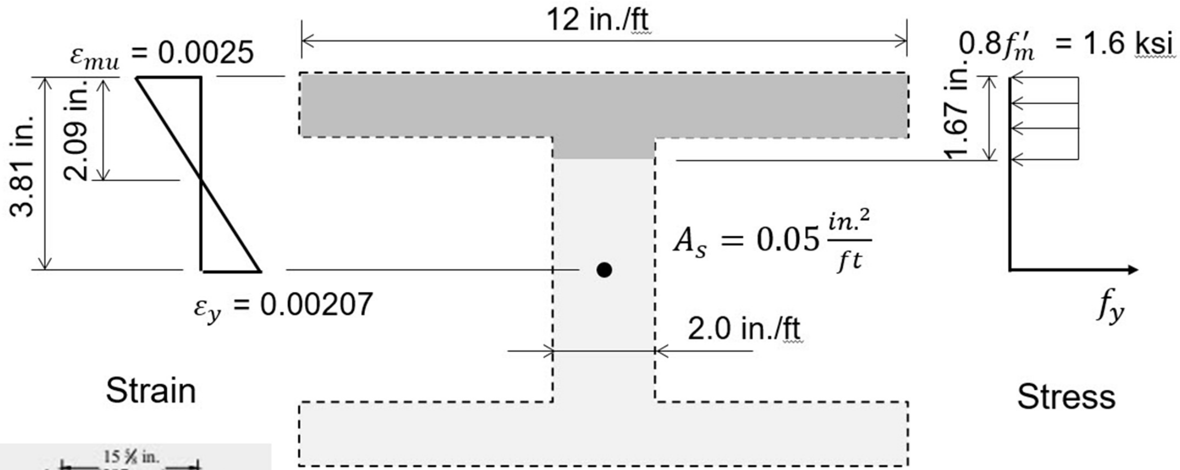
$$\phi P_n = 20.1\frac{k}{ft} \qquad \phi M_n = 5.97\frac{k \cdot ft}{ft}$$

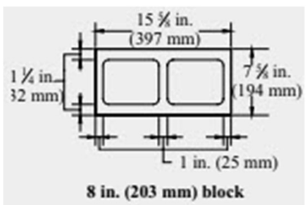
Interaction Diagram



Balanced

Example – 8 in. CMU bearing wall





web length = $\frac{8in.}{48in.}\frac{12in.}{ft} = 2.0\,\frac{in.}{ft}$

$$c = \frac{\varepsilon_{mu}}{\varepsilon_{mu} + \varepsilon_y} d = \frac{0.0025}{0.0025 + 0.00207} 3.81in. = 2.09in.$$

$$a = 0.8c = 0.8(2.09in.) = 1.67in.$$

Interaction Diagram

Below Balance

Tension, T $\quad T = f_y A_s$

Compression, C_m $\quad C_m = 0.8 f'_m b a$

Nominal Axial Strength, P_n $\quad P_n = C_m - T = 0.8 f'_m b a - A_s f_y$

Solve for a $\quad a = \dfrac{A_s f_y + P_n}{0.8 f'_m b}$

Nominal Moment Strength, M_n

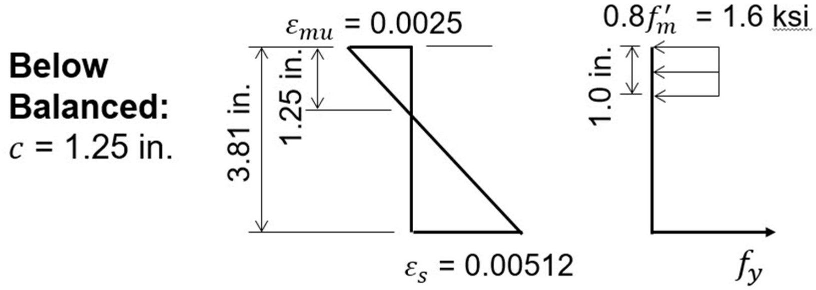
$$M_n = 0.8 f_m b a \left(\frac{t_{sp}}{2} - \frac{a}{2}\right) + A_s f_y \left(d - \frac{t_{sp}}{2}\right)$$

$$= \left(P_n + A_s f_y\right)\left(\frac{t_{sp} - a}{2}\right) + A_s f_y \left(d - \frac{t_{sp}}{2}\right)$$

Can solve for M_n if P_n is known

Below Balanced

Example – 8 in. CMU bearing wall



$$\phi P_n = 14.6\frac{k}{ft}$$
$$\phi M_n = 4.76\ \frac{k\cdot ft}{ft}$$

$$C_m = [0.8(2.0ksi)](1.0in.)\left(12\frac{in.}{ft}\right) = 19.2\frac{k}{ft}$$

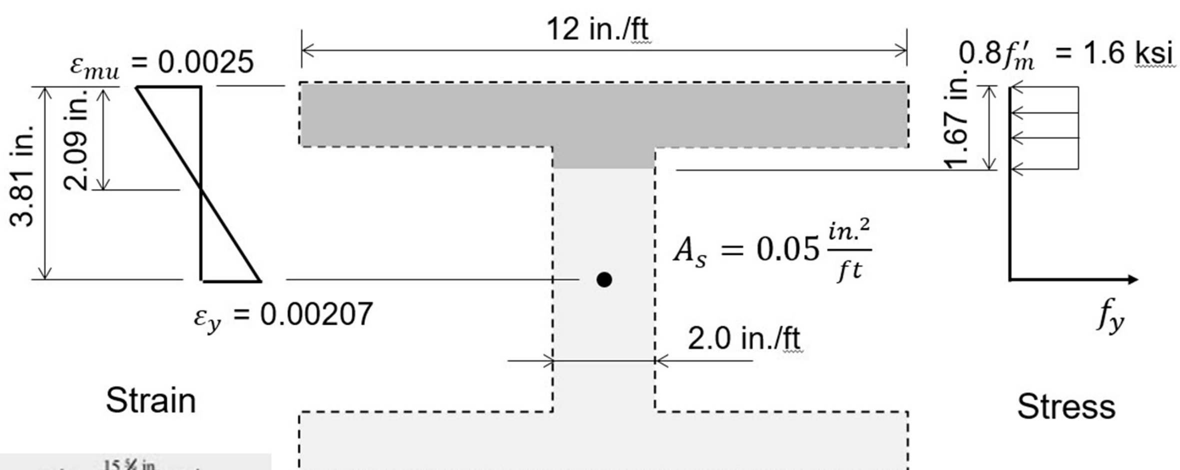
$$T = f_y A_s = 60ksi\left(0.05\frac{in.^2}{ft}\right) = 3.0\frac{k}{ft}$$

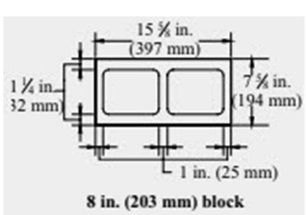
$$\phi P_n = 0.9(19.2 - 3.0)\frac{k}{ft} = 14.6\frac{k}{ft}$$

$$\phi M_n = 0.9\left[19.2\frac{k}{ft}\left(3.81 - \frac{1.0}{2}\right)in./(12)\right] = 4.76\ \frac{k\cdot ft}{ft}$$

Balanced

Example – 8 in. CMU bearing wall



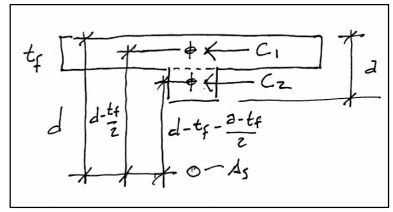


web length = $\frac{8in.}{48in.}\frac{12in.}{ft} = 2.0\frac{in.}{ft}$

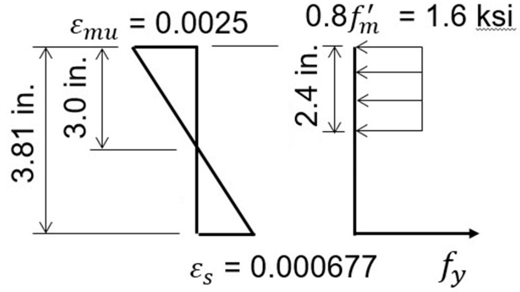
$$c = \frac{\varepsilon_{mu}}{\varepsilon_{mu} + \varepsilon_y}d = \frac{0.0025}{0.0025 + 0.00207}3.81in. = 2.09in.$$
$$a = 0.8c = 0.8(2.09in.) = 1.67in.$$

Above Balanced

Example – 8 in. CMU bearing wall



Above Balanced:
c = 3.0 in.



$$\phi P_n = 24.0\,\frac{k}{ft}$$
$$\phi M_n = 6.28\,\frac{k\cdot ft}{ft}$$

$$C_{m,fs} = [0.8(2.0ksi)](1.25in.)\left(12\,\frac{in.}{ft}\right) = 24.0\,\frac{k}{ft}$$
$$C_{m,web} = [0.8(2.0ksi)](2.4in. - 1.25in.)\left(2.0\,\frac{in.}{ft}\right) = 3.68\,\frac{k}{ft}$$

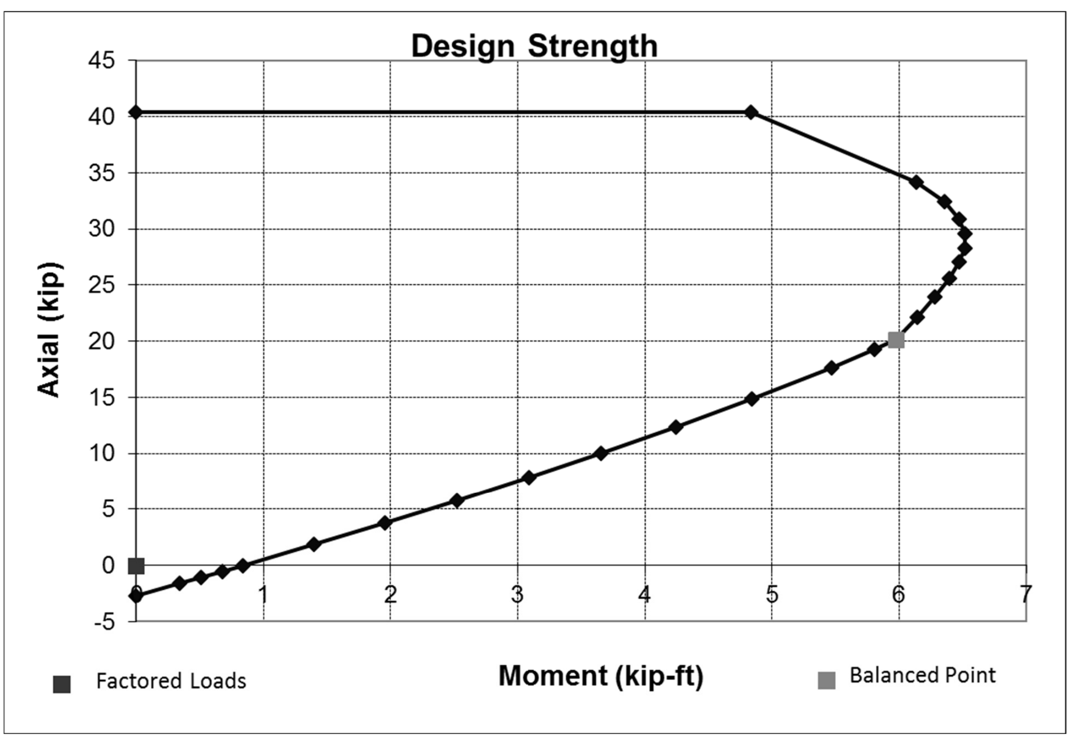
$$T = E_s\varepsilon_s A_s = 29000ksi(0.000677)\left(0.05\,\frac{in.^2}{ft}\right) = 0.98\,\frac{k}{ft}$$

$$\phi P_n = 0.9(24.0 + 3.68 - 0.98)\,\frac{k}{ft} = 24.0\,\frac{k}{ft}$$

$$\phi M_n = 0.9\left[24.0\,\frac{k}{ft}\left(3.81 - \frac{1.25}{2}\right)in. + 3.68\,\frac{k}{ft}\left(3.81 - 1.25 - \frac{2.4-1.25}{2}\right)in.\right]$$
$$= 6.28\,\frac{k\cdot ft}{ft}$$

Interaction Diagram

Example – 8 in. CMU bearing wall



Interaction Diagram

Example – 8 in. CMU bearing wall

Point	c (in.)	$C_{m,fs}$ (kip/ft)	$C_{m,web}$ (kip/ft)	T (kip/ft)	ϕP_n (kip/ft)	ϕM_n (kip-ft/ft)
$a = d$	4.76	24.0	8.2	0	29.0	6.52
$c = d$	3.81	24.0	5.8	0	26.8	6.45
	3.00	24.0	3.7	1.0	24.0	6.28
Balanced	2.09	24.0	1.3	3.0	20.1	5.97
a = 1.25 in.	1.56	24.0	0	3.0	18.9	5.73
	1.25	19.2	0	3.0	14.6	4.77
	1.0	15.4	0	3.0	11.1	3.93
	0.8	12.3	0	3.0	8.4	3.22
	0.6	9.2	0	3.0	5.6	2.47
	0.4	6.1	0	3.0	2.8	1.68
Pure Moment	0.195	3.0	0	3.0	0	0.84

Interaction Diagram

Solid vs. Partial Grout